



# Syllabus



Principles and Applications of Systems Biology

Vassily Hatzimanikatis  
September 2025



# Assistants



Omar Keshk



David Liaskos



Luca Milazzo

# Syllabus (approximate)

Date	Lecture	Exercise
11.09	Introduction to Systems Biology, FBA, Objective functions, Growth trade off (FBA1)	Q/A - Environment Setup
18.09	Phenotypic phase plane, alternative flux distributions (FBA2) Practical (Software; Simulation exercises) + dFBA	Q/A - Excercises
25.09	Data integration of Stoichiometric models; Practical exercises	Q/A - Excercises
02.10	Thermodynamics flux balance analysis, and GCM (TFA2)	Q/A - Excercises
09.10	Enzyme kinetics/Large-scale modelling + Kinetic modeling of cancer metabolism	Q/A
16.10	Signal transduction systems – Network reconstruction	Q/A
30.10	Signal transduction systems - Dynamics	Q/A
06.11	ME-models	Q/A
13.11	Microbial communities	Q/A
20.11	Other topics (Whole-cell and whole-body models)	Q/A
27.11	Work on projects	Q/A
04.12	<i>Final Presentations</i>	
11.12	<i>Final Presentations</i>	
18.12		

## Brief course description

The course introduces and develops the key concepts from **systems biology** and **systems engineering** in the context of **complex biological networks**.

The lectures elaborate on **techniques** and **methods** to **model** and **analyze** complex biological problems.

*(Focus on Metabolism – and some signaling!)*

Examples and projects apply the model-based and systems engineering integration and analysis of big data from biological systems.

## TOPICS

- Mathematical and computational analysis of metabolic reaction networks
- Analysis of metabolomics and bioenergetics data in the context metabolic networks
- Mathematical and computational analysis of protein expression
- Methods and technologies for the analysis of signaling networks
- Computational models of microbial communities
- Interpretation and analysis of single cell data
- Mathematical modeling of spatial effects in biological systems

# METHODS

- Metabolic Flux balance analysis (FBA)  
**(Linear programming)**
- Thermodynamics based flux balance analysis (TFA) of metabolic networks  
**(Mixed-integer linear programming).**
- Kinetic models  
**(Ordinary differential equations)**
- Metabolic control analysis  
**(Local and global sensitivity analysis)**
- Stochastic simulation algorithm (SSA) and Particle based simulation methods  
**(Stochastic simulation)**
- Parameter estimation for biological systems  
**(System identification methods)**

## RECOMMENDATIONS

The building of working groups will make it possible for people with partial knowledge in these fields to contribute depending on their formation.

## COURSES

### *SV courses:*

- Dynamical systems in biology BIO-341
- Numerical analysis MATH-251

### *ChemE courses:*

- Dynamics and kinetics CH-310
- Biochemical engineering ChE-311
- Bioreactor modelling and simulation ChE-320
- Numerical methods ChE-312

# RECOMMENDATIONS

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- Bioreactor modelling and simulation ChE-320
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## Important concepts to start the course

- For the computational exercises, MATLAB® and PYTHON will be the platforms of choice.
- An introductory session on the platforms and software used is part of the course.

## Learning Outcomes

By the end of the course, the student must be able to:

- Formulate mass balances of reaction networks
- Solve mass balance equations using linear programming solvers
- Construct kinetic models of biological reactions
- Create and analyze stochastic models of biological reactions
- Analyze papers on modeling and analysis of biological networks
- Assess / Evaluate alternative methods for the study of biological networks

## Transversal skills

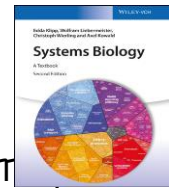
- Plan and carry out activities in a way which makes optimal use of available time and other resources.
- Access and evaluate appropriate sources of information.
- Summarize an article or a technical report.
- Demonstrate the capacity for critical thinking
- Negotiate effectively within the group.

# Literature

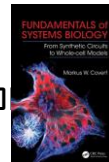
- Foundations of System Biology, Edited by Hiraaki Kitano. MIT Press 2001 (classic reading)



- Systems Biology, a textbook, Edda Klipp



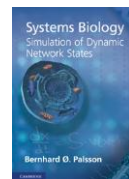
- Fundamentals of Systems Biology: From Circuits to Whole-Cell Models, by Markus Covert



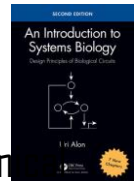
- Systems Biology : Constraint-Based Reconstruction and Analysis



- Systems Biology: Simulation of Dynamic Network States



An Introduction to Systems Biology: Design Principles of Biological Circuits, by Uri Alon. Chapman and Hall/CRC 2006

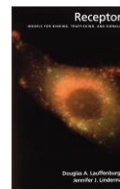


- Computational Modeling of Genetic and Biochemical Networks, by James M. Bower and Hamid Bolouri. Bradford 2004

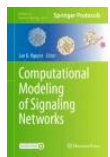
- Receptors: Models for Binding, Trafficking, and Signaling by Douglas A. Lauffenburger, Jennifer Linderman



- Cellular Signal Processing: An Introduction to the Molecular Mechanisms of Signal Transduction



- Computational Modeling of Signaling Networks



# Grading

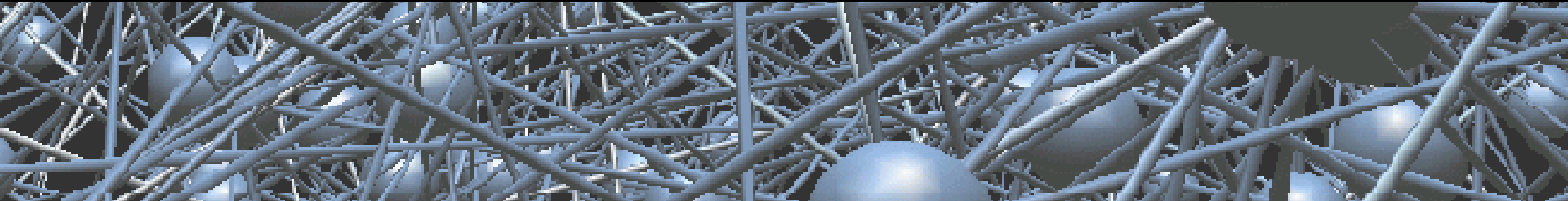
- Project based evaluation:
  - Analyze and reproduce the results of a paper (A paper will be assigned to each group)
- TAs will be available to solve the questions during exercise sessions
- One grade per group
- Final presentation 100%

# Groups and Project presentations

- Groups:
  - 2-3 People
  - Send one e-mail per group to [david.liaskos@epfl.ch](mailto:david.liaskos@epfl.ch) with the full names of all group members until the 27<sup>th</sup> Sep
  - If you are not part of a group by that time, we will assign you to a group
- Project presentation:
  - After forming a group, a paper will be assigned to you
  - Goal: Replicate the key results from the article
  - Propose extensions to the work presented



# Introduction into Systems Biology



Principles and Applications of Systems Biology

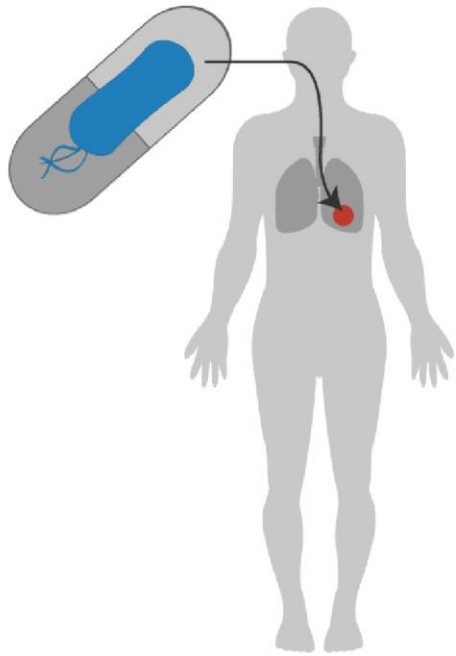
Vassily Hatzimanikatis

September 2025

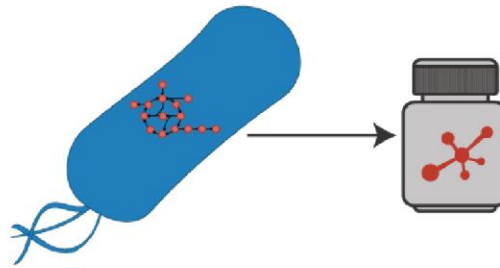
# Systems Biology

Understanding biology on a systems level:

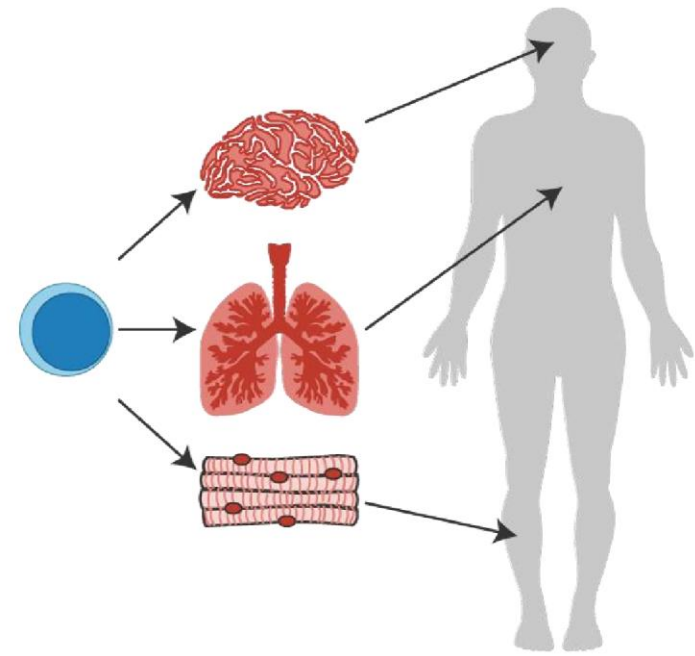
# Systems Biology - Why?



Biosensors



Biofactories



Tissue engineering

# Complex Systems

Imagine a system consists of N **agents** that make **individual decisions** and **interact** with each other.

If the behavior of the system **cannot be deduced from the behavior of the individual agents** but can only deduced from **all the participating agents and their interactions** this system is complex.

Let A,B,C and D be departments in a company that perform a special task and interact with each other.

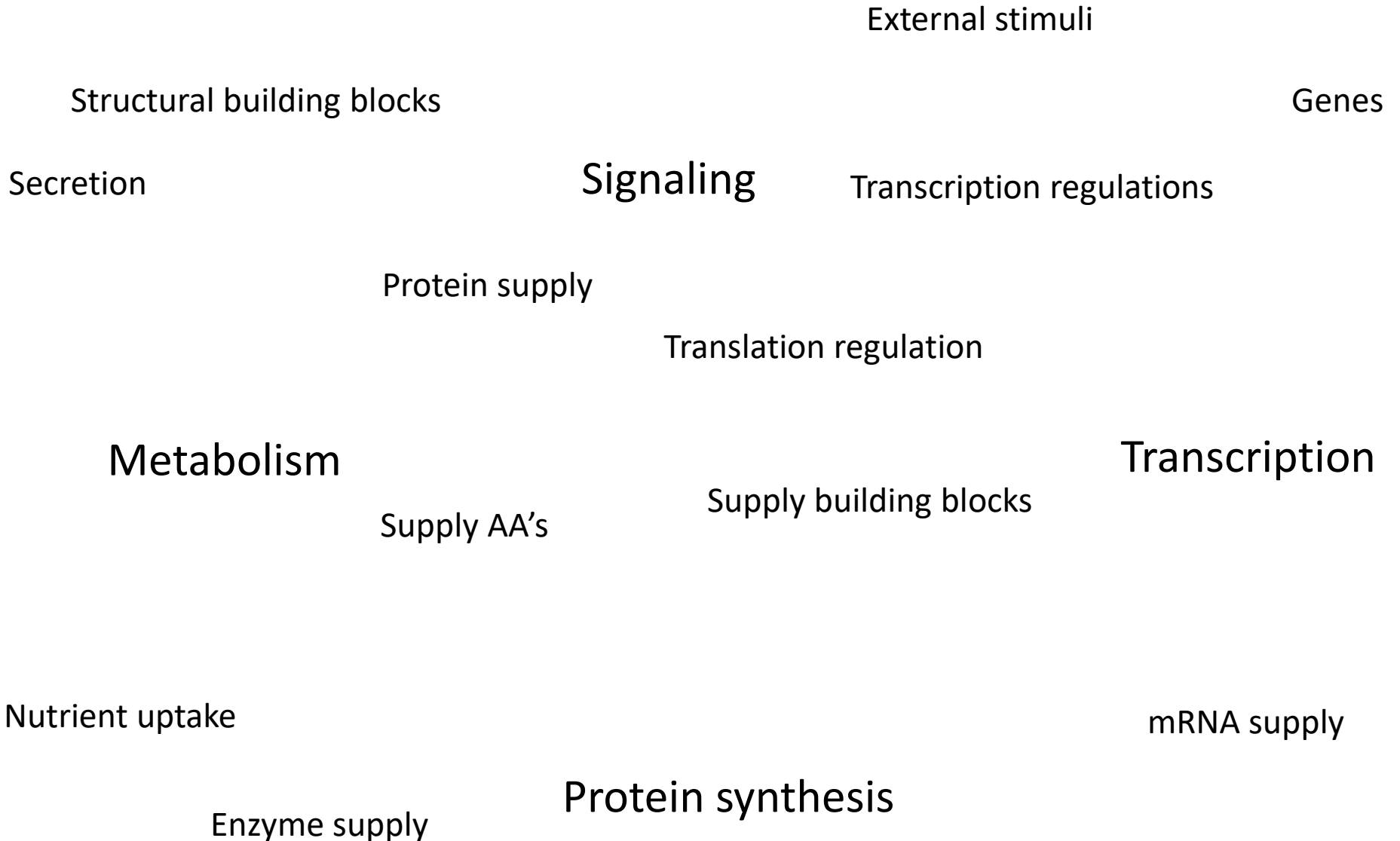
The overall performance of this company will not be determined by how fast the individual departments perform their task with a given input resources but how these departments interchange resources and information.

# The cell a complex system

Similar departments exist in the cell:

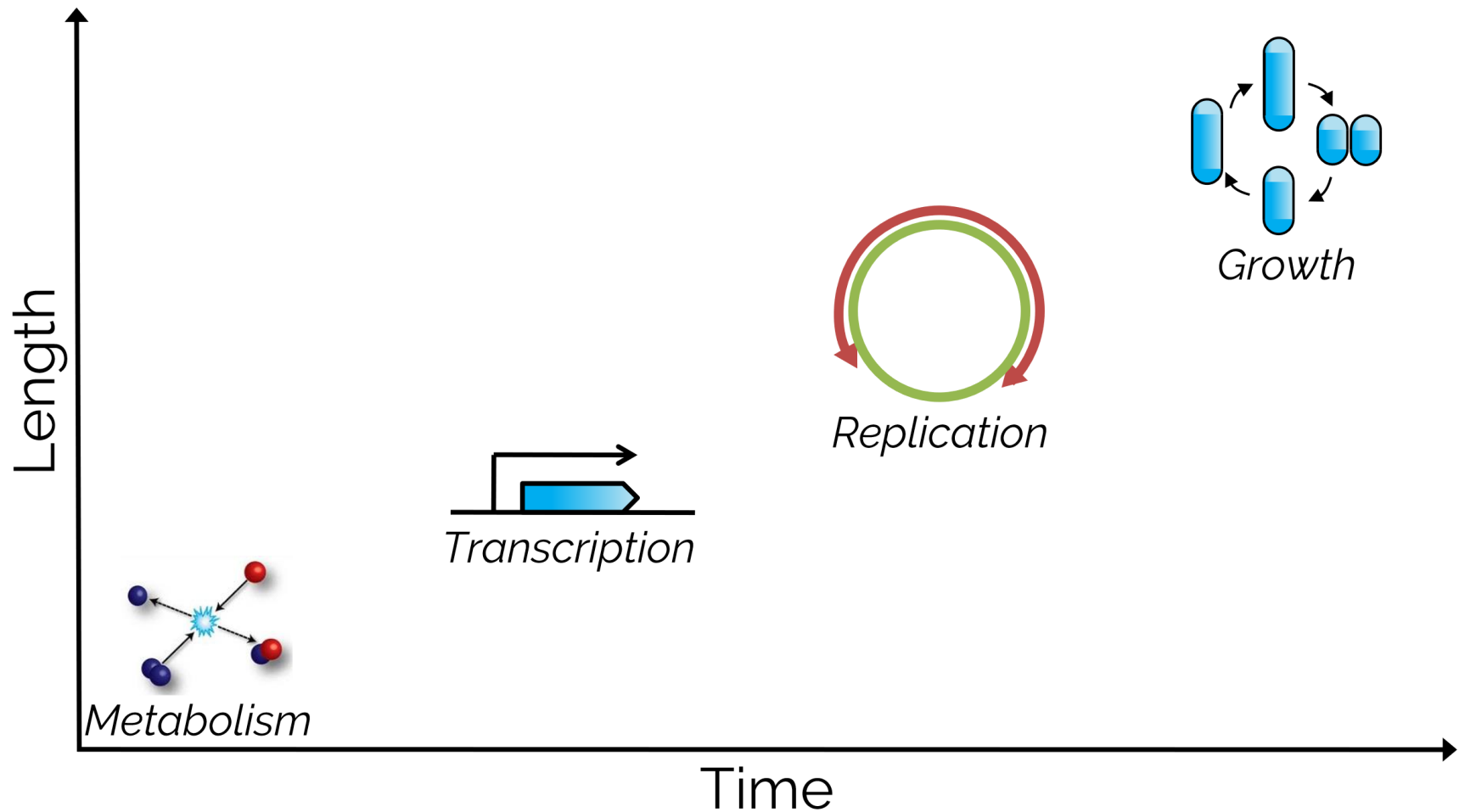
# The cell a complex system

# The cell a complex system

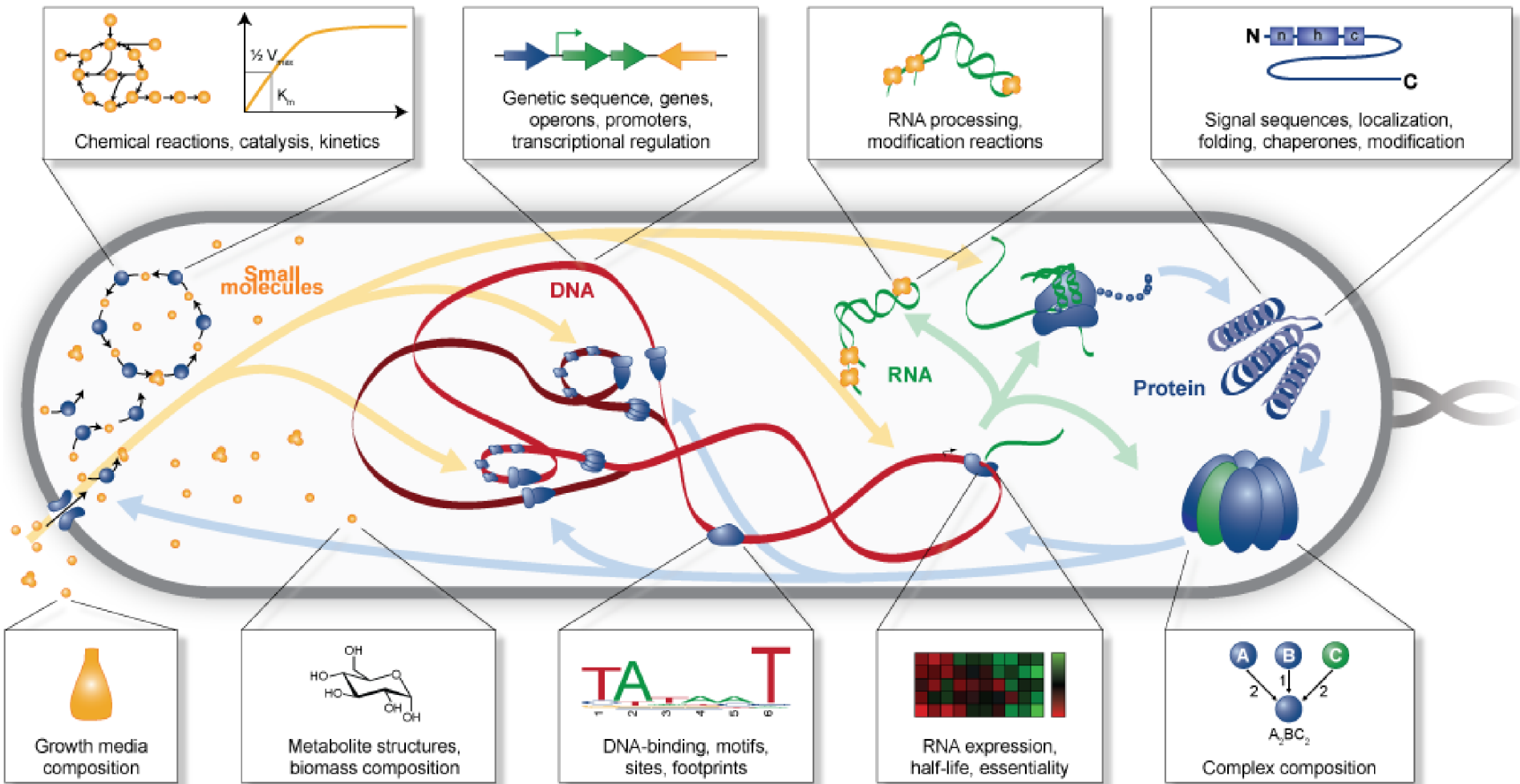


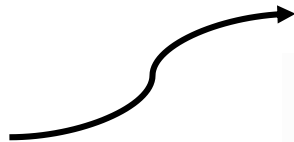


# Complexity in time

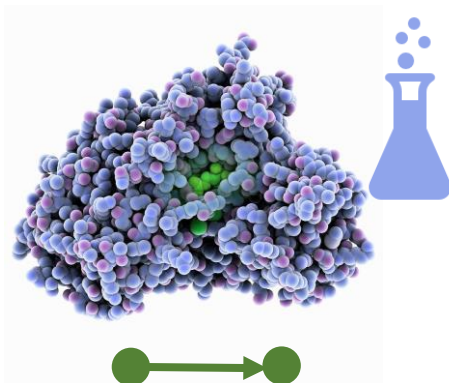


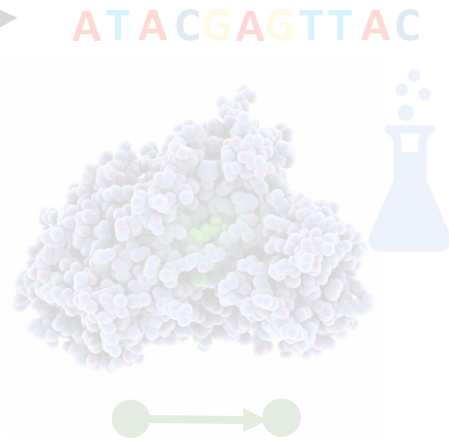
# Integration of data and systems

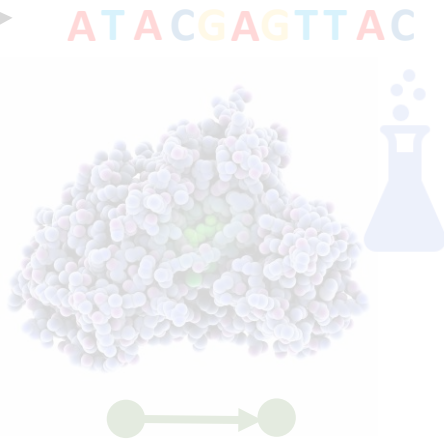




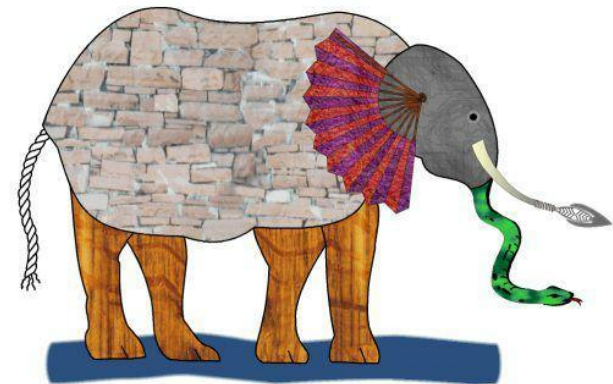
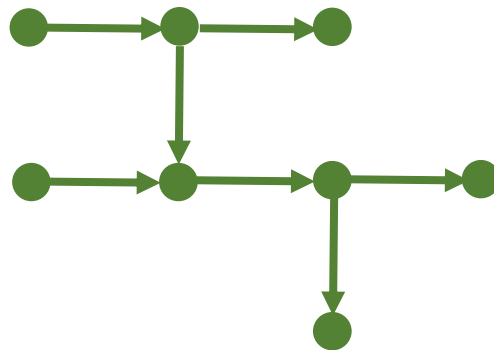
ATACGAGTTAC





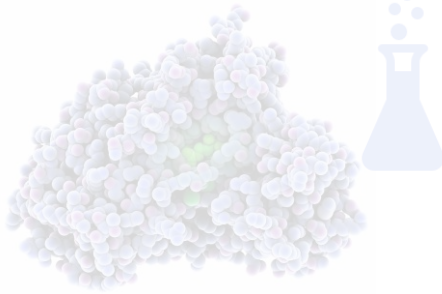


The whole system is different from the individual parts and includes **emergent properties**

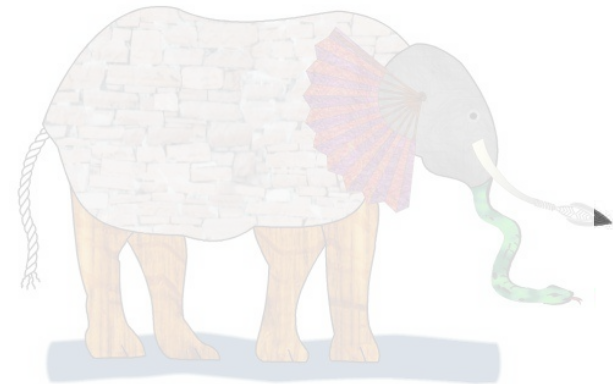
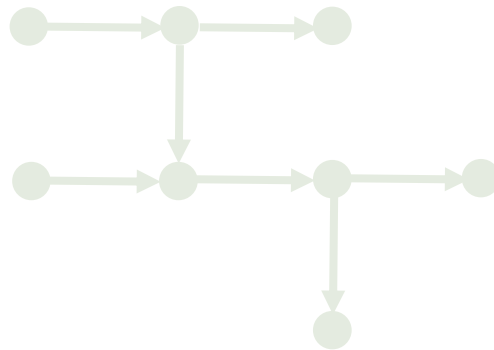
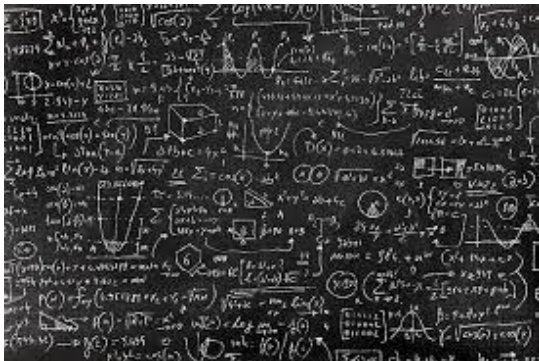




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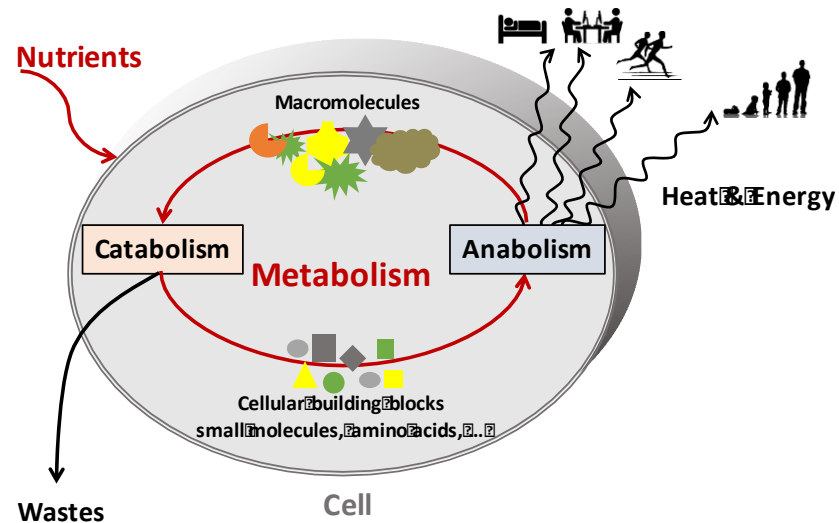


A mathematical model can represent the **parts** and **interactions** in the system



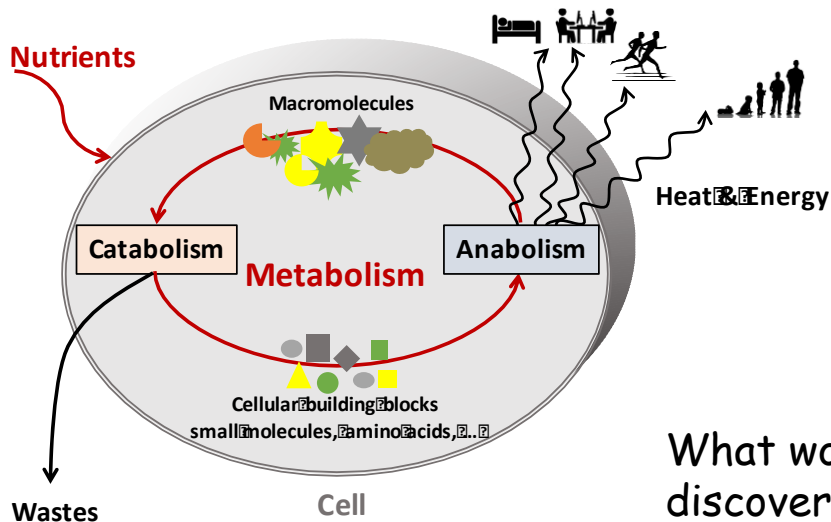
# Systems Biology of Metabolism

Ensemble of biochemical reactions that occur in living organisms in order to maintain life



# Metabolism

## Understanding metabolism



What can we currently measure?

What does omics data mean individually?

*Genomics*: what may happen in cell  
*Transcriptomics*: what can happen in cell  
*Proteomics*: What should happen in cell  
*Metabolomics*: What is happening in cell

How do we integrate the data to understand cell as a system?

What would you need to measure if you wanted to discover the causes of disease?

What are the mechanisms of drugs?

Why pathogens have large metabolic versatility and flexibility?

Design new therapies?

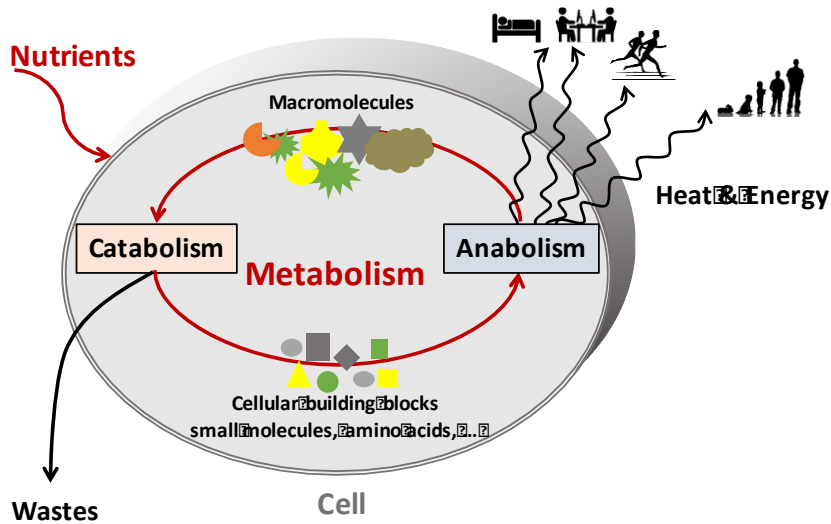
How metabolism is re-programmed in critical condition? Starvation?

How are the metabolic pathways different in micro-organisms?

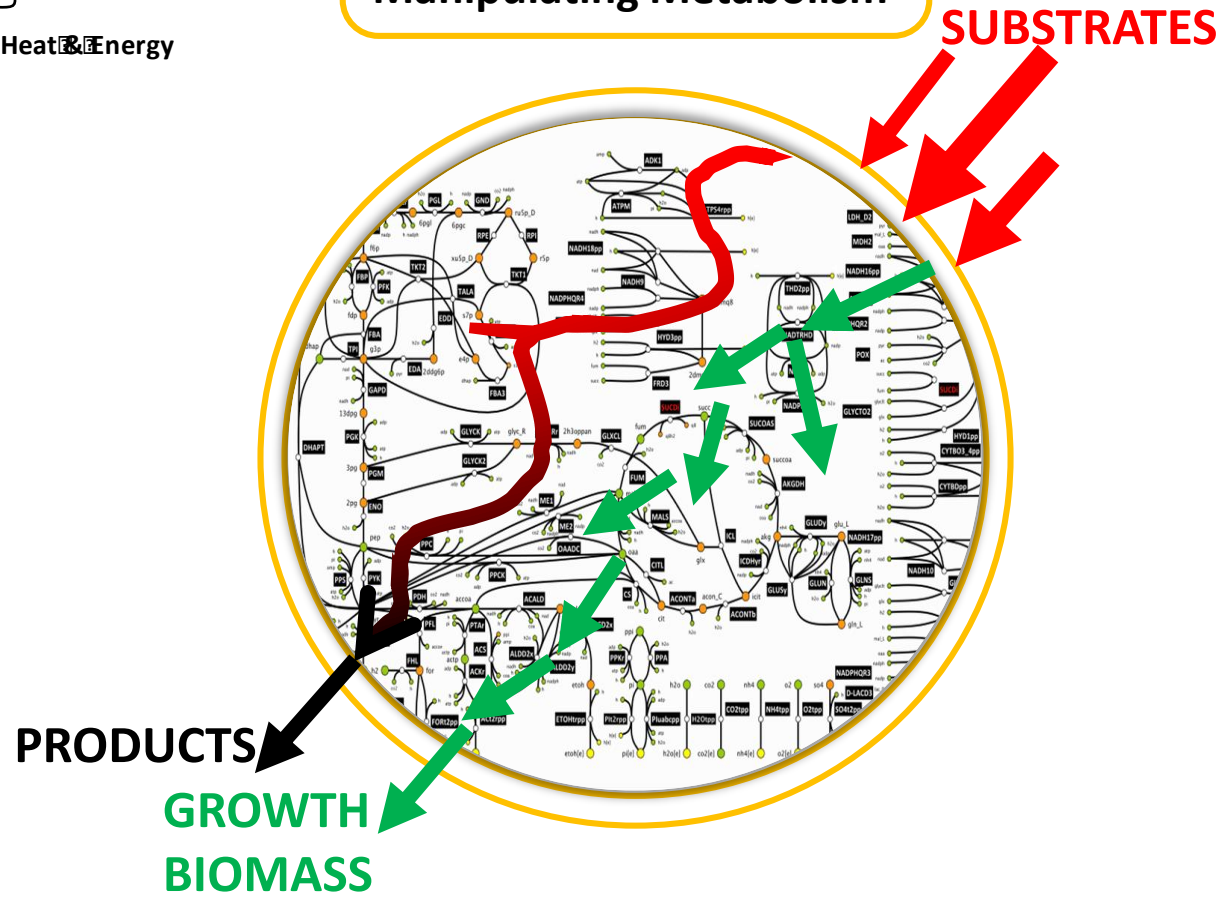
**re-engineer organisms for new purposes? Cell factories?**

# Metabolism

Understanding metabolism



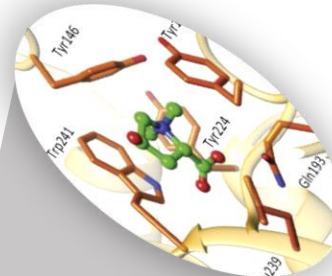
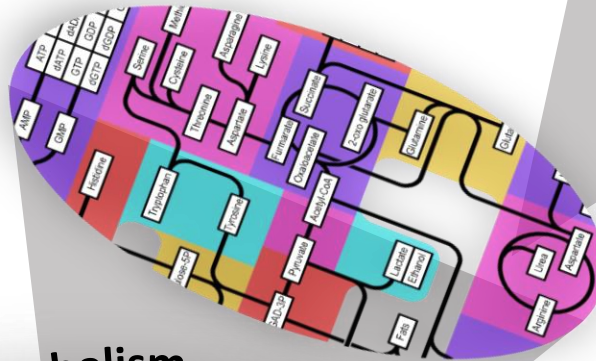
Manipulating Metabolism





# System components

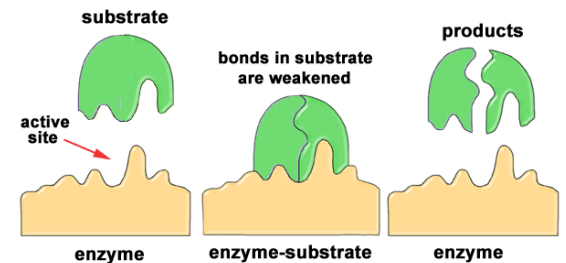
(Biochemical pathways, ex. glycolysis)



~5800 enzymatic reactions



proteins that accelerate the rate of biochemical reaction

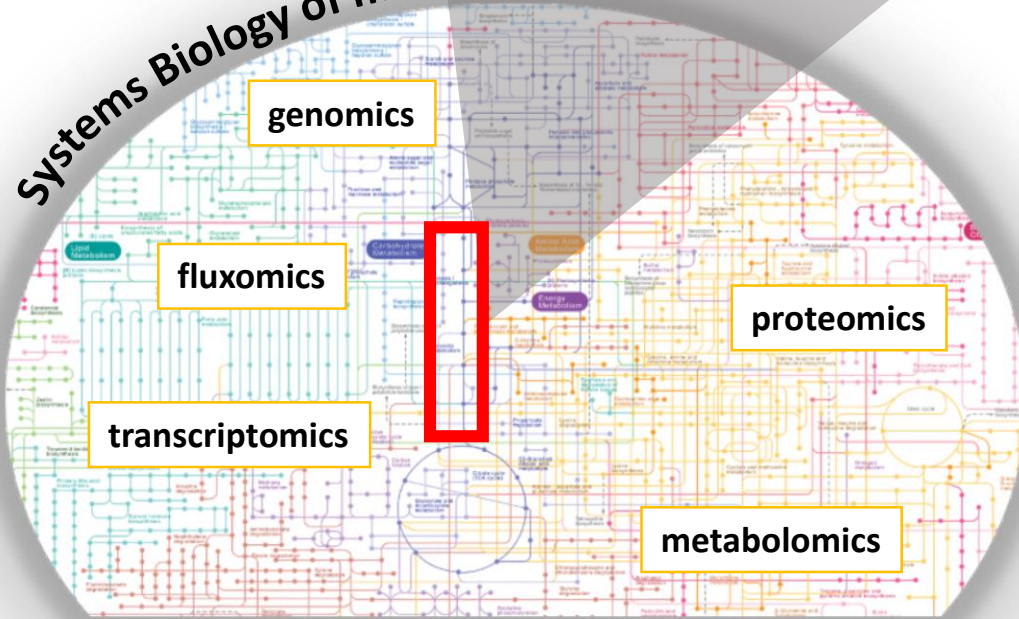


Biochemistry toolbox



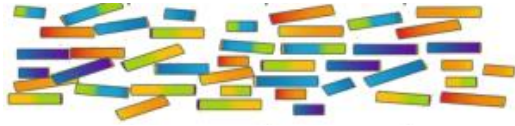
Genome sequencing

Systems Biology of metabolism

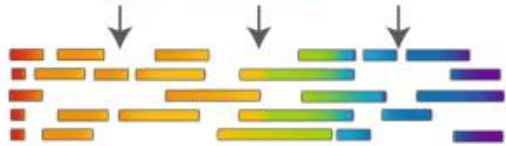


# From sequenced genome to enzymatic functions

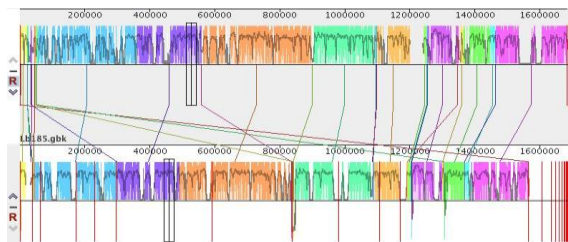
Sequence quality and trimming/filtering  
(FastQC, Trimmomatic)



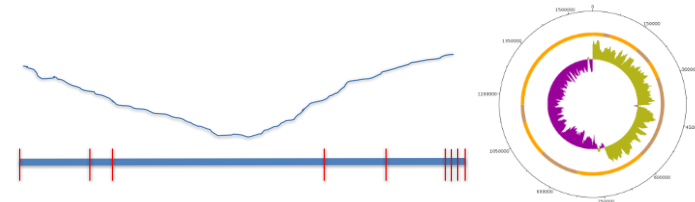
Genome assembly  
(SPAdes)



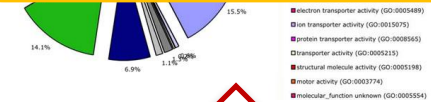
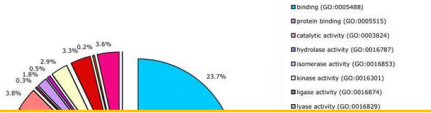
Contigs re-ordering and trimming  
(Artemis, Blast, Mauve)



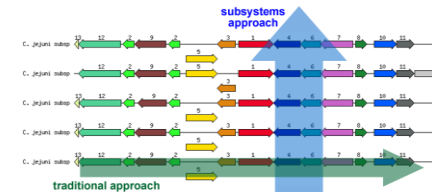
Assembly Validation  
(Read coverage, GC-skew/content)



## Metabolic functions



Annotation  
(RAST, subsystems approach)





# Enzyme Classification

## **Enzyme Commission number (EC number)**

Back in 1955, in the International Congress of Biochemistry in Brussels, the enzyme nomenclature scheme was developed and named: Enzyme Commission (EC) number

EC numbers do not specify enzymes (protein sequences) but enzyme-catalyzed reactions (functions).

If different enzymes (for instance from different organisms) catalyze the same reaction, they receive the same EC number.

# EC number (i.j.k.l)

- **i.** general type of chemistry

<b>EC 1</b> : Oxidoreductases	To catalyze oxidation/reduction reactions
<b>EC 2</b> : Transferases	Transfer of a functional group from one substance to another
<b>EC 3</b> : Hydrolases	Formation of two products from a substrate by hydrolysis
<b>EC 4</b> : Lyases	Non-hydrolytic addition or removal of groups from substrates
<b>EC 5</b> : Isomerases	Intramolecule rearrangement
<b>EC 6</b> : Ligases	Join together two molecules by synthesis of new C-O, C-S, C-N or C-C bonds

# EC number (i.j.k.l)

- **i.** general type of chemistry
- **j.** functional group being acted upon

## **EC 1** : Oxidoreductases

To catalyze oxidation/reduction reactions

- 1.1** act on the CH-OH group of donors
- 1.2** act on the aldehyde or oxo group of donors
- 1.3** act on the CH-CH group of donors
- 1.4** act on the CH-NH<sub>2</sub> group of donors
- 1.5** act on CH-NH group of donors
- 1.6** act on NADH or NADPH
- 1.7** act on other nitrogenous compounds as donors
- 1.8** act on a sulfur group of donors
- 1.9** act on a heme group of donors
- 1.10** act on diphenols and related substances as donors
- 1.11** act on peroxide as an acceptor
- 1.12** act on hydrogen as donors
- 1.13** act on single donors with incorporation of molular oxygen
- 1.14** act on paired donors with incorporation of molular oxygen
- 1.15** act on superoxide radicals as acceptors
- 1.16** oxidize metal ions
- 1.17** act on CH or CH<sub>2</sub> groups
- 1.18** act on iron-sulfur proteins as donors
- 1.19** act on reduced flavodoxin as a donor
- 1.20** act on phosphorus or arsenic in donors
- 1.21** act on X-H and Y-H to form an X-Y bond
- 1.97** includes other oxidoreductases

# EC number (i.j.k.l)

- **i.** general type of chemistry
- **j.** functional group being acted upon
- **k.** cofactors the enzyme uses or specific information on the chemistry of the enzyme action

**EC 1** : Oxidoreductases

To catalyze oxidation/reduction reactions

**1.1** act on the CH-OH group of donors

**1.1.1** With NAD or NADP as acceptor

**1.1.2** With a cytochrome as acceptor

**1.1.3** With oxygen as acceptor

**1.1.4** With a disulfide as acceptor

**1.1.5** With a quinone or similar compound as acceptor

**1.1.9** With a copper protein as acceptor

**1.1.98** With other, known, acceptors

**1.1.99** With other acceptors

# EC number (i.j.k.l)

- **i.** general type of chemistry
- **j.** functional group being acted upon
- **k.** cofactors the enzyme uses or specific information on the chemistry of the enzyme action
- **l.** specific substrates participating in the reaction

**EC 1** : Oxidoreductases

To catalyze oxidation/reduction reactions

**1.1** act on the CH-OH group of donors

**1.1.1** With NAD or NADP as acceptor

**1.1.1.1**: alcohol dehydrogenase

**1.1.1.2**: alcohol dehydrogenase (NADP+)

**1.1.1.3**: homoserine dehydrogenase

**1.1.1.4**: (R,R)-butanediol dehydrogenase

**1.1.1.5**: Transferred to \* 1.1.1.303 and \* 1.1.1.304

**1.1.1.6**: glycerol dehydrogenase

**1.1.1.7**: propanediol-phosphate dehydrogenase

**1.1.1.8**: glycerol-3-phosphate dehydrogenase (NAD+)

**1.1.1.9**: D-xylulose reductase

**1.1.1.10**: L-xylulose reductase

**1.1.1.11**: D-arabinitol 4-dehydrogenase

**1.1.1.12**: L-arabinitol 4-dehydrogenase

**1.1.1.13**: L-arabinitol 2-dehydrogenase

**1.1.1.14**: L-iditol 2-dehydrogenase

**1.1.1.15**: D-iditol 2-dehydrogenase

**1.1.1.16**: galactitol 2-dehydrogenase

**1.1.1.17**: mannitol-1-phosphate 5-dehydrogenase

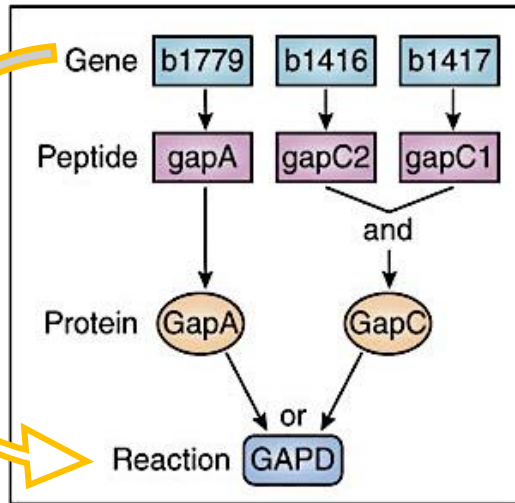
**1.1.1.18**: inositol 2-dehydrogenase

# Genome Scale Metabolic Models

✓ Systems Analysis of Metabolism

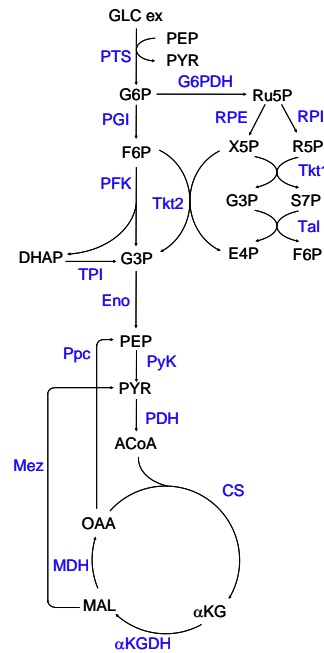
Sequenced and Annotated Genome

## Functional Annotation



## Metabolic Reactions -> Glycolysis pathway

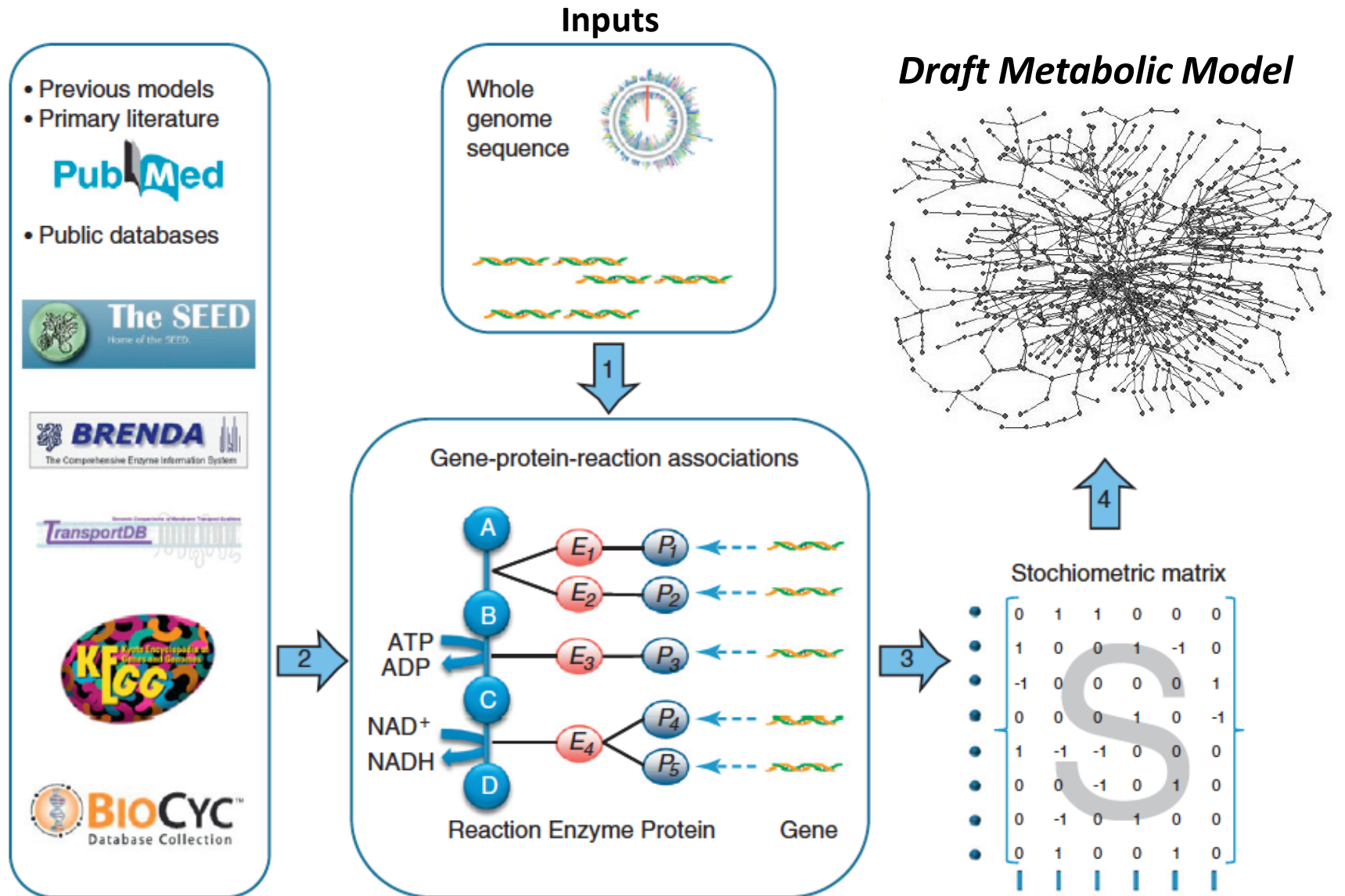
Abbreviation	Glycolytic reactions
HEX1	[c]GLC + ATP → G6P + ADP+ H
PGI	[c]G6P ↔ F6P
PFK	[c]ATP + F6P → ADP + FDP + H
FBA	[c]FDP ↔ DHAP + G3P
TPI	[c]DHAP ↔ G3P
<b>GAPD</b>	<b>[c]G3P + NAD + PI ↔ 13DPG + H + NADH</b>
PGK	[c]13DPG + ADP ↔ 3PG + ATP
PGM	[c]3PG ↔ 2PG
ENO	[c]2PG ↔ H <sub>2</sub> O + PEP
PYK	[c]ADP + H + PEP → ATP + PYR



1000s of reactions for a single species

- Computers can keep this information "in mind" and analyze it in various ways.
- How to digitalize this information and show the interactions between metabolites and reactions?
- **Interaction Matrices** summarize any kind of interaction between elements of a system.

# Genome Scale Model Reconstruction



# Genome Scale Model Reconstruction



# Genome Online Databases

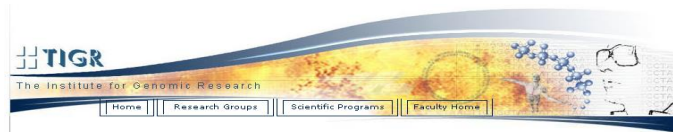
## Genome Databases



Entrez Gene



<http://www.ncbi.nlm.nih.gov/entrez/query.fcgi?CMD=search&DB=gene>



<http://pathema.tigr.org/tigr-scripts/CMR/CmrHomePage.cgi>



<http://genomesonline.org/index2.htm>

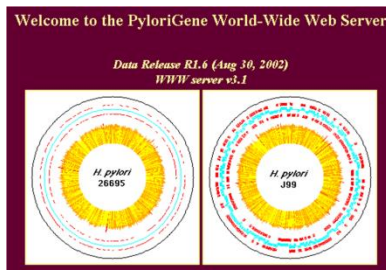


<http://vega.sanger.ac.uk/index.html>



<http://cmr.tigr.org/tigr-scripts/CMR/CmrHomePage.cgi>

## Organism-specific databases



**H. pylori:** <http://genolist.pasteur.fr/PyloriGene/>



Encyclopedia of *Escherichia coli* K-12 Genes and Metabolism

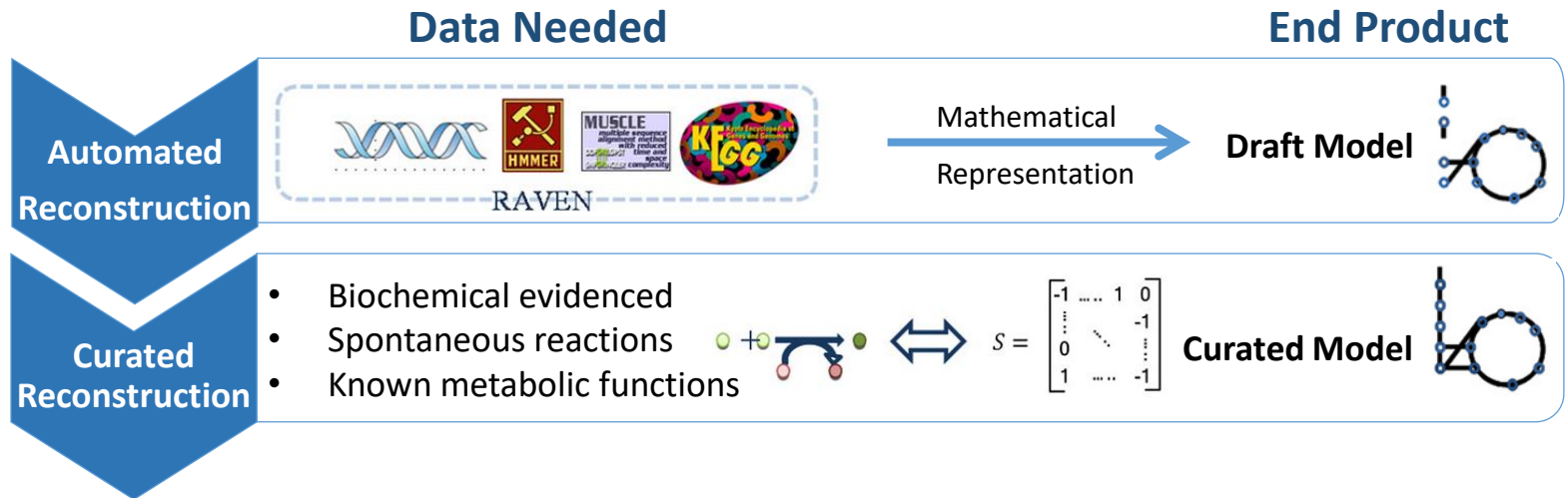
<http://ecocyc.org/>



**Saccharomyces Genome Database**

<http://www.yeastgenome.org/>

# Genome Scale Model Reconstruction



# Biochemical databases

- **Enzyme databases:**

- KEGG:

<http://www.genome.jp/kegg/>

- BRENDA:

<http://www.brenda-enzymes.info/>

KEGG: Kyoto Encyclopedia of Genes and Genomes

A grand challenge in the post-genomic era is to complete computer representation of the cell, the organism, and the biosphere, which will enable computational prediction of higher-level complexity of cellular processes and organism behaviors from genomic- and molecular information. Towards this end we have been developing a bioinformatics resource named KEGG as part of the research projects of the Kanoh Laboratory in the Bioinformatics Center of Kyoto University and the Human Genome Center of the University of Tokyo.

- **Main entry point to the KEGG web service**
  - KEGG2 KEGG Table of Contents Update notes Help
- **Data-oriented entry points**
  - KEGG Atlas Global maps of cell/organism functions
  - KEGG PATHWAY Pathway maps and pathway modules
  - KEGG BRITE Functional hierarchies and ontologies
  - KEGG GENES Genomes, genes, proteins, and orthologs
  - KEGG LIGAND Chemical compounds, drugs, glycans, and reactions
- **Organism-specific entry points**
  - KEGG Organism: Select [Organism] [Go] (example) hsa
  - KEGG DISEASE Gene/molecule based disease information resource
  - KEGG DRUG Chemical structure based drug information resource
  - KEGG KEGG PATHWAY Knowledge base for pathway reconstruction
  - KEGG COMPOUND Knowledge base for biochemical compounds
  - KEGG REACTION Knowledge base for biochemical reactions
  - KAAS KEGG automatic annotation server

Copyright 1995-2008 Kanoh Laboratory

BRENDA  
The Comprehensive Enzyme Information System

Latest BRENDA update: June 2008

Nomenclature	Reaction & Specificity	Functional Parameters
Enzyme Names EC Number Common Recommended Name Systematic Name Synonyms CAS Registry Number	Pathway Catalysed Reaction Reaction Type Natural Substrates and Products Substrates and Products Substrates Natural Substrate Products Natural Product Inhibitors	Ion Value K <sub>i</sub> Value K <sub>50</sub> Value p Value Turnover Number Specific Activity pH Optimum pH Range Temperature Optimum Temperature Range
Isolation & Preparation Purification Cloned Renatured Crystallization	Cofactors Metallofactors Activating Compounds Ligands	Organism-related information Organism Source Tissue Localization Protein-Specific Search
Stability pH Stability Temperature Stability General Stability Organic Solvent Stability Oxidation Stability Storage Stability	Enzyme Structure Sequence/ SwissProt link 3D-Structure/ PDB link Molecular Weight Subunits Posttranslational Modification	Disease & References Disease References
		Application & Engineering Engineering Application

Webmaster: Maurice Scheer  
m.scheer@brenda.de

For access to all features of the website Javascript must be activated. Frames enabled and Java (at least version 1.4) has to be installed

- Both databases are great resources for biochemical reactions, but there information are organism-unspecific!

- **Transport database:**

- Transport DB:

<http://www.membranetransport.org/>

- Transport Classification Database

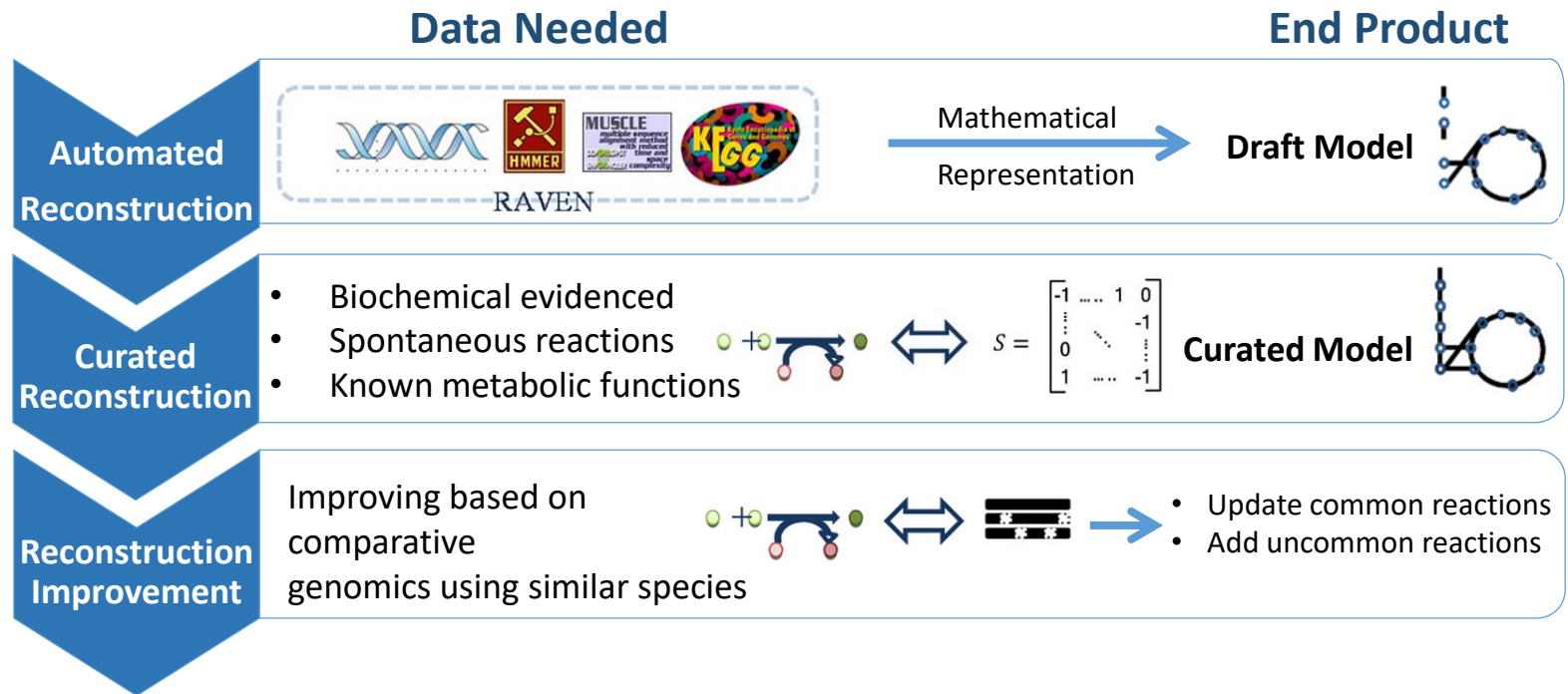
Genomic Comparisons of Membrane Transport Systems  
**TransportDB**

All Organisms  
Eukaryotes

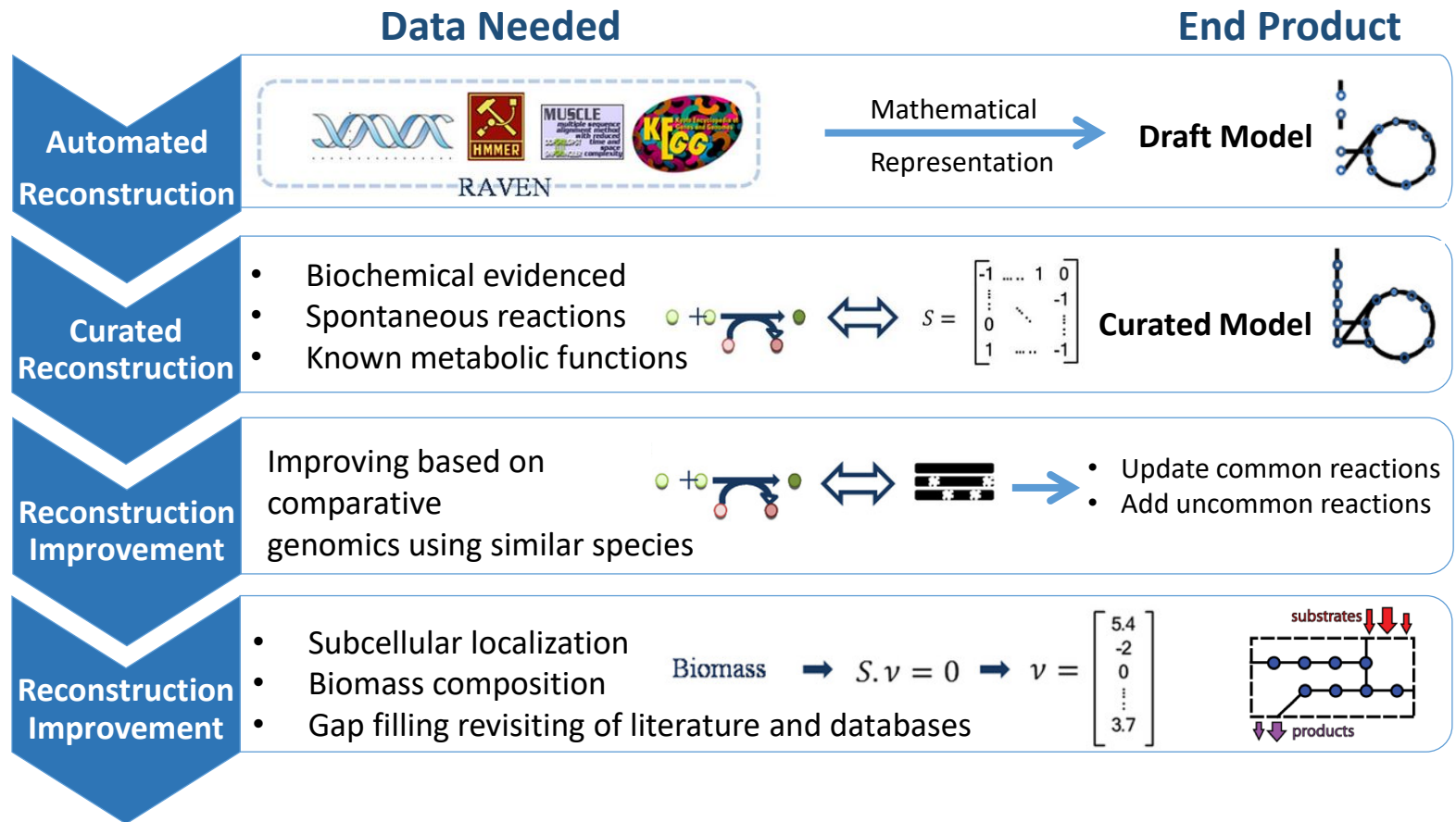
**TCDB**  
UCSD

<http://www.tcdb.org/>

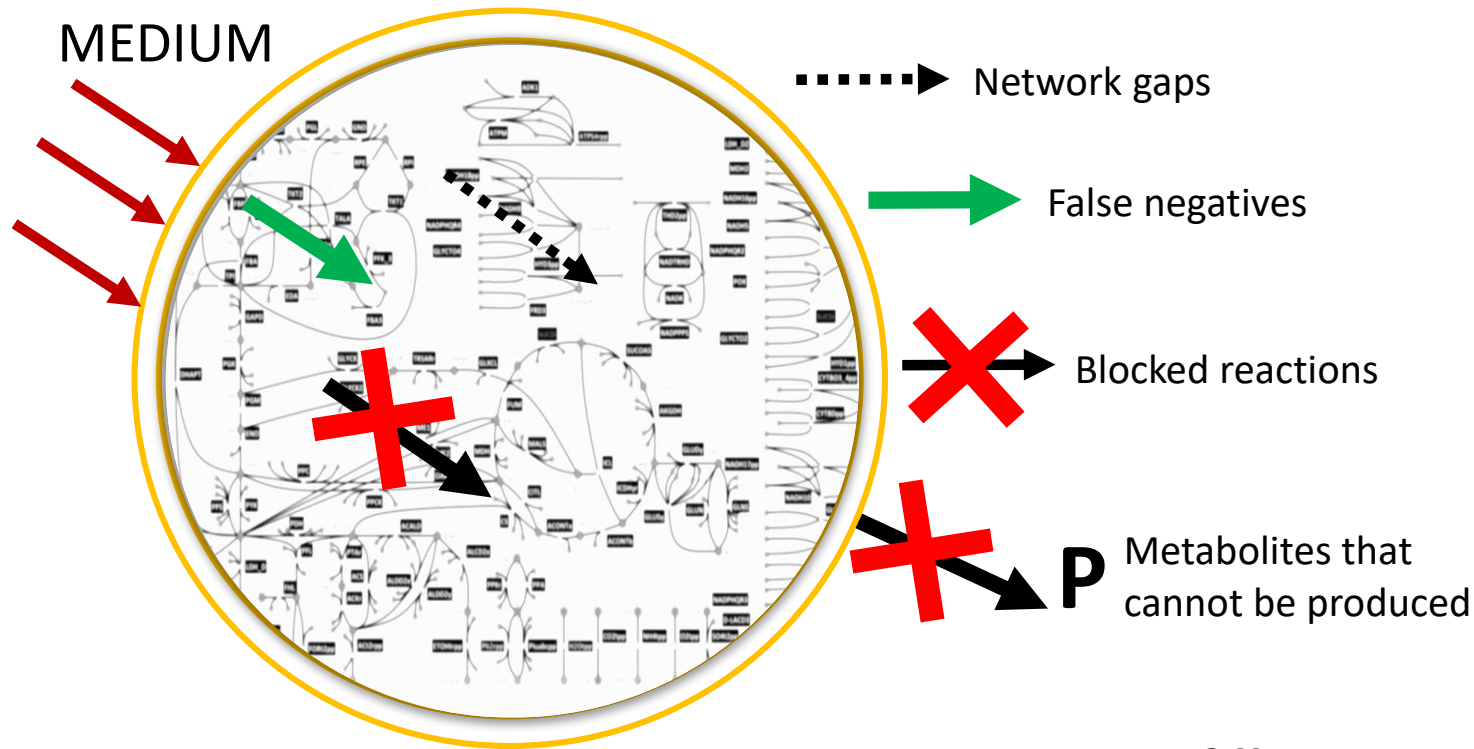
# Genome Scale Model Reconstruction



# Genome Scale Model Reconstruction



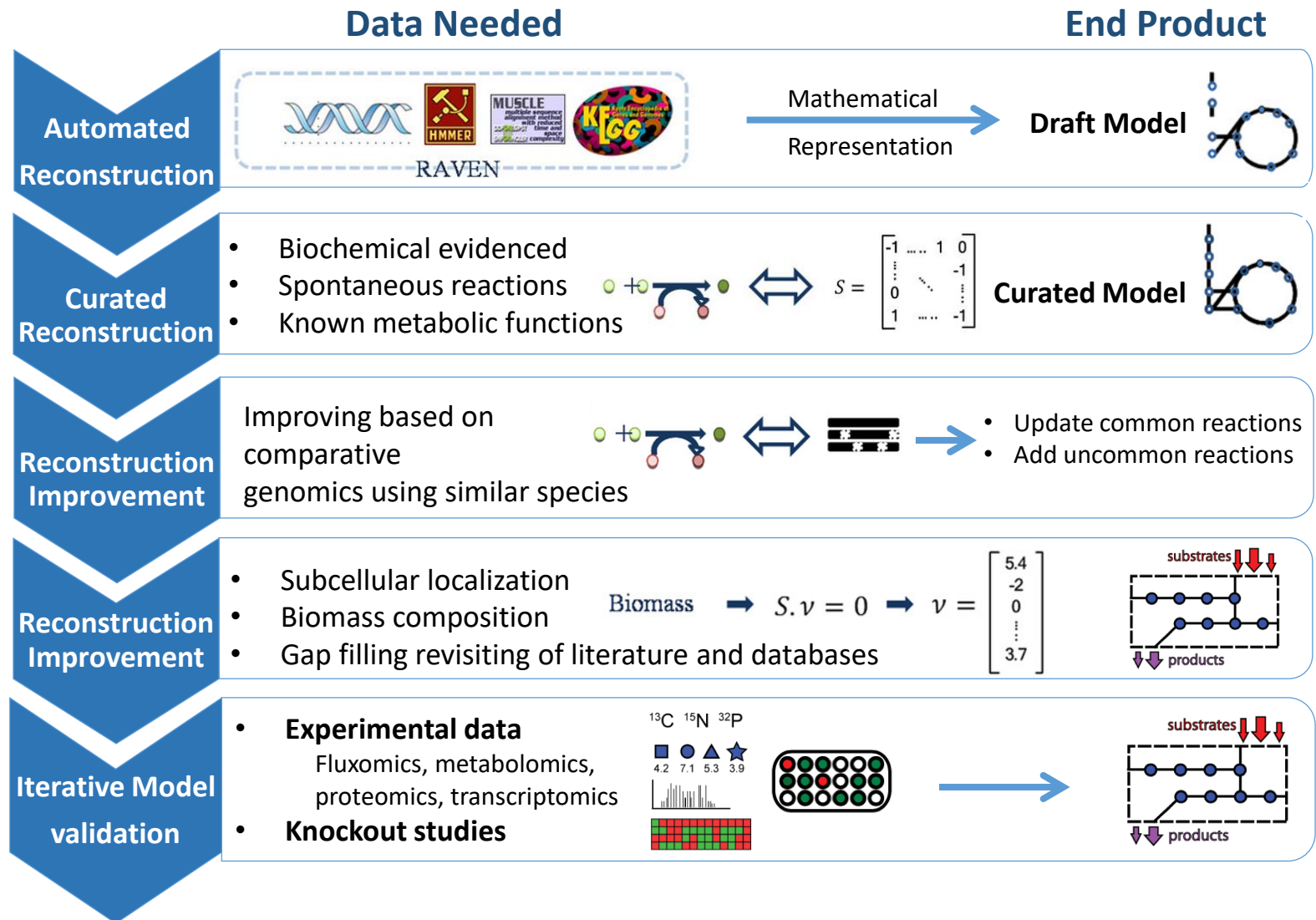
# Knowledge Gaps in metabolic networks



## Systematic Gap-filling using:

- ✓ Other annotation platforms
- ✓ *Close organism*
- ✓ KEGG database
- ✓ ATLAS of Biochemistry

# Genome Scale Model Reconstruction



# Genome Scale Metabolic Models

Genome Scale Models are driven from **sequenced genome**

- They started with the reconstruction of metabolic network of microbes
- Now they exist for several mice strains and human

## Industrially relevant organisms

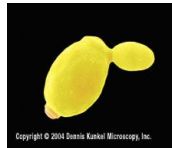
*E. coli*

- 2712 Reactions
- 1516 Genes



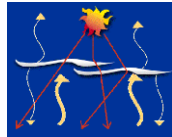
*S. cerevisiae*

- 1402 Reactions
- 910 Genes



*M. barkeri*

- 619 Reactions
- 692 Genes



*G. sulfurreducens*

- 608 Reactions
- 588 Genes



*B. subtilis*

- 1020 Reactions
- 844 Genes



## Pathogens

*S. aureus*

- 640 Reactions
- 619 Genes



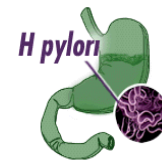
*S. typhimurium*

- 2545 Reactions
- 1271 Genes



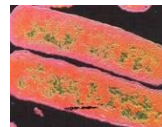
*H. pylori*

- 558 Reactions
- 341 Genes



*H. influenzae*

- 472 Reactions
- 376 Genes



*M. tuberculosis*

- 939 Reactions
- 661 Genes



## Mammalian cells

*H. sapiens*

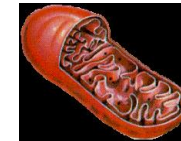
- 10600 Reactions
- 2248 Genes



Human

Mitochondria

- 218 Reactions



Red blood cell

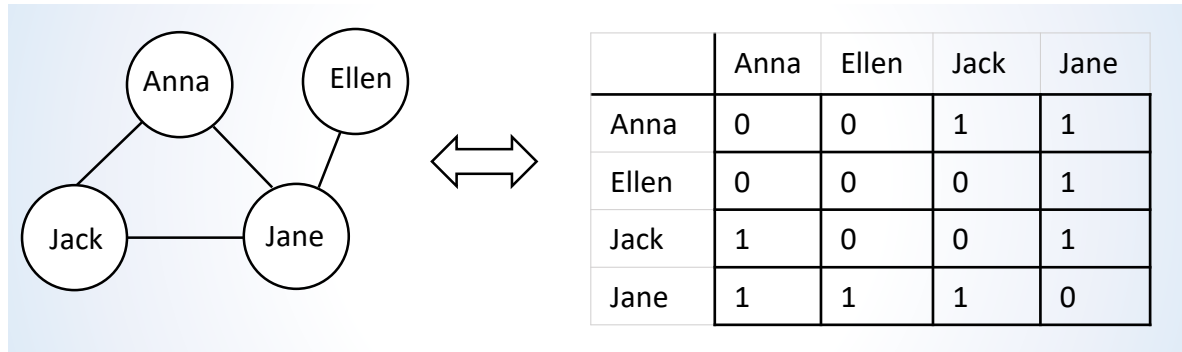
- 39 Reactions



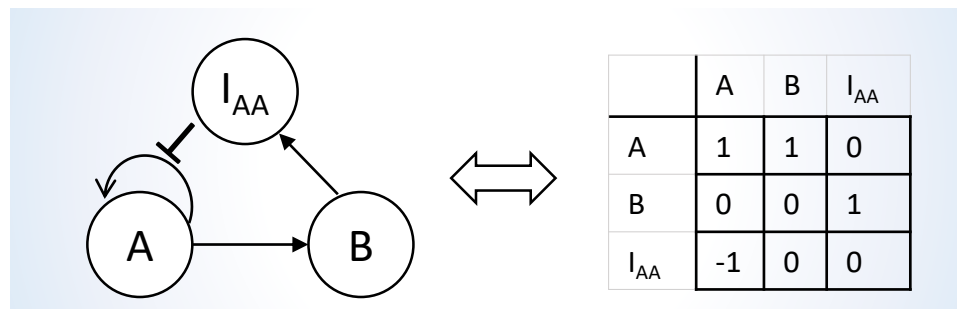
# Interaction Matrices

Represents the **relationship** between all the **elements** (constituent) of a system.

## Example 1: social networks



## Example 2: gene Regulatory Network

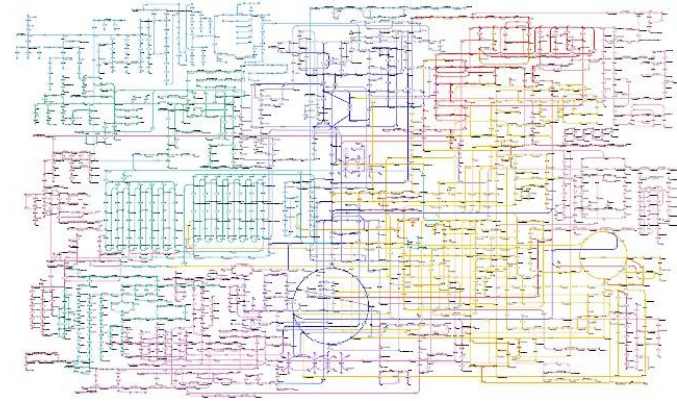
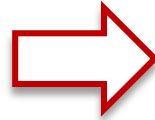


# Stoichiometric Matrix

Represents the relationship between *all the metabolites* in *all the reactions* in a metabolic network

**Metabolic Reactions**

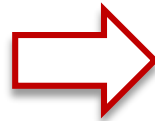
Abbreviation	Glycolytic reactions
HEX1	$[c]GLC + ATP \rightarrow G6P + ADP + H$
PGI	$[c]G6P \leftrightarrow F6P$
PFK	$[c]ATP + F6P \rightarrow ADP + FDP + H$
FBA	$[c]FDP \leftrightarrow DHAP + G3P$
TPI	$[c]DHAP \leftrightarrow G3P$
GAPD	$[c]G3P + NAD + PI \leftrightarrow 13DPG + H + NADH$
PGK	$[c]13DPG + ADP \leftrightarrow 3PG + ATP$
PGM	$[c]3PG \leftrightarrow 2PG$
ENO	$[c]2PG \leftrightarrow H_2O + PEP$
PYK	$[c]ADP + H + PEP \rightarrow ATP + PYR$



Reactions →

	HEX1	PGI	PFK	FBA	TPI	GAPD	PGK	PGM	ENO	PYK
ATP	-1	0	-1	0	0	0	1	0	0	1
GLC	-1	0	0	0	0	0	0	0	0	0
ADP	1	0	1	0	0	0	-1	0	0	-1
G6P	1	-1	0	0	0	0	0	0	0	0
H	1	0	1	0	0	1	0	0	0	-1
F6P	0	1	-1	0	0	0	0	0	0	0
FDP	0	0	1	-1	0	0	0	0	0	0
DHAP	0	0	0	1	-1	0	0	0	0	0
G3P	0	0	0	1	1	-1	0	0	0	0
NAD	0	0	0	0	0	-1	0	0	0	0
PI	0	0	0	0	0	-1	0	0	0	0
13DPG	0	0	0	0	0	1	-1	0	0	0
NADH	0	0	0	0	0	1	0	0	0	0
3PG	0	0	0	0	0	0	1	-1	0	0
2PG	0	0	0	0	0	0	0	1	-1	0
PEP	0	0	0	0	0	0	0	0	1	-1
H <sub>2</sub> O	0	0	0	0	0	0	0	0	1	0
PYR	0	0	0	0	0	0	0	0	0	1

↑ Metabolites



Reactions

Metabolites

$$\begin{bmatrix} 1 & 0 & \dots & 1 \\ 0 & -1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \dots & -1 \end{bmatrix}$$

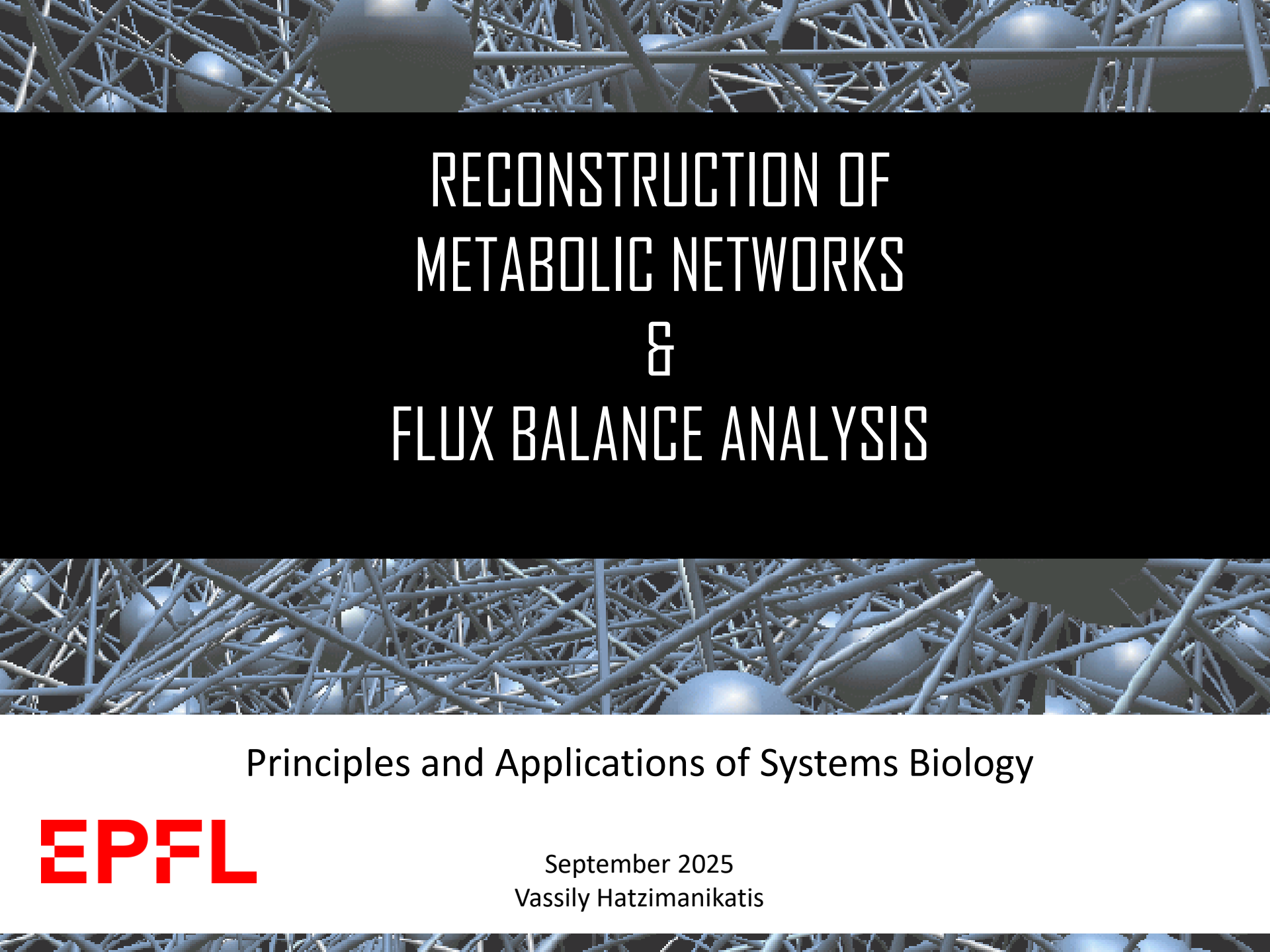
# Closing Remarks

**Genome-scale metabolic model is a platform that**

agrees with experimentally observed data

allows testing hypotheses and answer metabolically relevant questions

allows generating new hypothesis for experimental validation



# RECONSTRUCTION OF METABOLIC NETWORKS & FLUX BALANCE ANALYSIS

Principles and Applications of Systems Biology

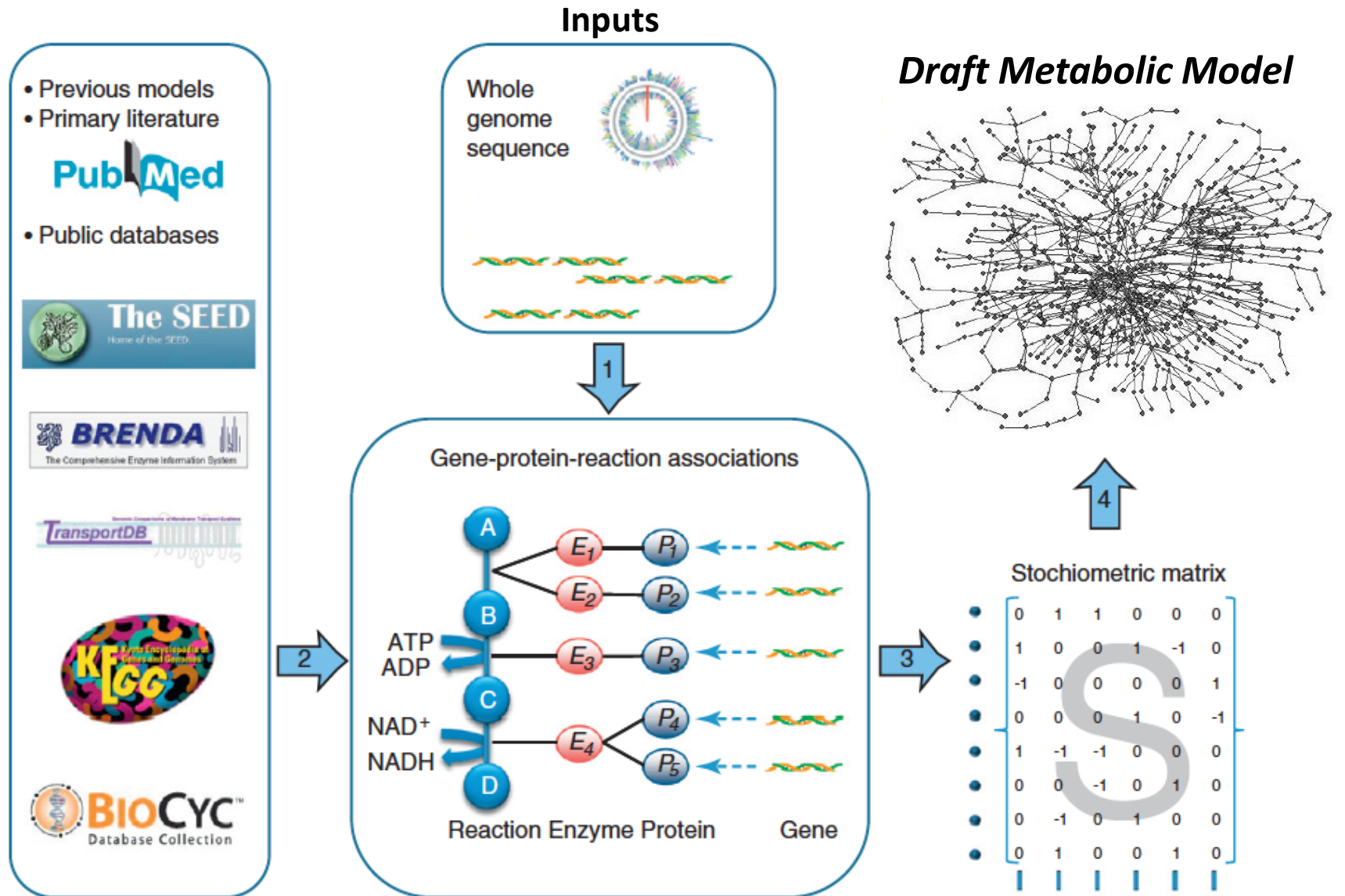
**EPFL**

September 2025  
Vassily Hatzimanikatis

# LECTURE OBJECTIVES

- GENERAL CONCEPTS OF MASS BALANCES IN METABOLIC NETWORKS
- METABOLIC NETWORK RECONSTRUCTION:
  - GENERAL WORKFLOW
  - BASIC CONCEPTS
- INTRODUCTION TO FLUX BALANCE ANALYSIS
  - LINEAR PROGRAMMING & OPTIMIZATION IN METABOLIC NETWORKS
- *WHAT IS THE OPTIMAL GROWTH OF BACTERIUM?*

# Genome Scale Model Reconstruction

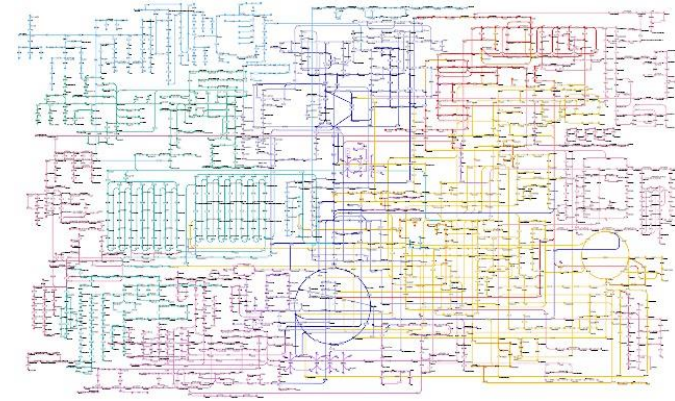
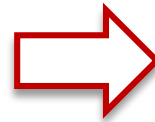


# Stoichiometric Matrix

Represents the relationship between *all the metabolites* in *all the reactions* in a metabolic network

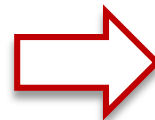
**Metabolic Reactions**

Abbreviation	Glycolytic reactions
HEX1	$[c]GLC + ATP \rightarrow G6P + ADP + H$
PGI	$[c]G6P \leftrightarrow F6P$
PFK	$[c]ATP + F6P \rightarrow ADP + FDP + H$
FBA	$[c]FDP \leftrightarrow DHAP + G3P$
TPI	$[c]DHAP \leftrightarrow G3P$
GAPD	$[c]G3P + NAD + PI \leftrightarrow 13DPG + H + NADH$
PGK	$[c]13DPG + ADP \leftrightarrow 3PG + ATP$
PGM	$[c]3PG \leftrightarrow 2PG$
ENO	$[c]2PG \leftrightarrow H_2O + PEP$
PYK	$[c]ADP + H + PEP \rightarrow ATP + PYR$



Reactions →

	HEX1	PGI	PFK	FBA	TPI	GAPD	PGK	PGM	ENO	PYK
ATP	-1	0	-1	0	0	0	1	0	0	1
GLC	-1	0	0	0	0	0	0	0	0	0
ADP	1	0	1	0	0	0	-1	0	0	-1
G6P	1	-1	0	0	0	0	0	0	0	0
H	1	0	1	0	0	1	0	0	0	-1
F6P	0	1	-1	0	0	0	0	0	0	0
FDP	0	0	1	-1	0	0	0	0	0	0
DHAP	0	0	0	1	-1	0	0	0	0	0
G3P	0	0	0	1	1	-1	0	0	0	0
NAD	0	0	0	0	0	-1	0	0	0	0
PI	0	0	0	0	0	-1	0	0	0	0
13DPG	0	0	0	0	0	1	-1	0	0	0
NADH	0	0	0	0	0	1	0	0	0	0
3PG	0	0	0	0	0	0	1	-1	0	0
2PG	0	0	0	0	0	0	0	1	-1	0
PEP	0	0	0	0	0	0	0	0	1	-1
H <sub>2</sub> O	0	0	0	0	0	0	0	0	1	0
PYR	0	0	0	0	0	0	0	0	0	1



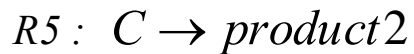
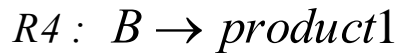
Reactions

Metabolites

$$\begin{bmatrix} 1 & 0 & \dots & 1 \\ 0 & -1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \dots & -1 \end{bmatrix}$$

# Mass Balances

## Metabolic Reactions



**S** matrix 

		Reactions				
		R1	R2	R3	R4	R5
Metabolites	Substrate	-1	0	0	0	0
	A	1	-1	-1	0	0
	B	0	1	0	-1	0
	C	0	0	1	0	-1
	Product 1	0	0	0	1	0
	Product 2	0	0	0	0	1

How much **substrates** ( $d_{\text{substrate}}/dt$ ) for how much **products** ( $d_{\text{p1}}/dt$ )?

# Fluxes

## Metabolic Reactions

$R1 : \text{Substrate} \rightarrow A$

$R2 : A \rightarrow B$

$R3 : A \rightarrow C$

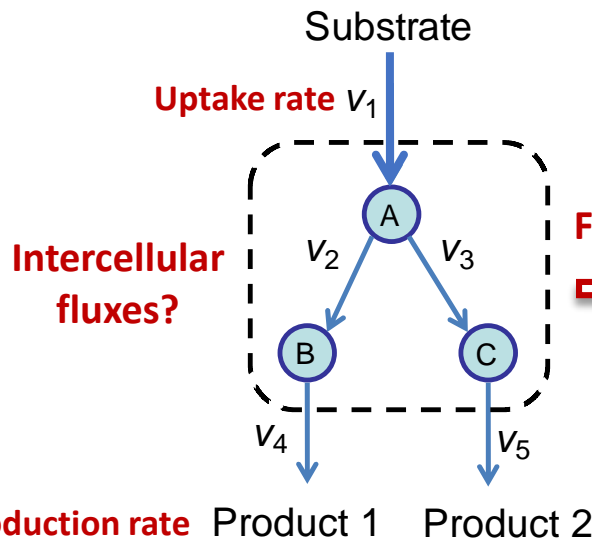
$R4 : B \rightarrow \text{product1}$

$R5 : C \rightarrow \text{product2}$

S matrix 

	Reactions				
	R1	R2	R3	R4	R5
Substrate	-1	0	0	0	0
A	1	-1	-1	0	0
B	0	1	0	-1	0
C	0	0	1	0	-1
Product1	0	0	0	1	0
Product2	0	0	0	0	1

How much **substrates** ( $d_{\text{substrate}}/dt$ ) for how much **products** ( $d_{p1}/dt$ )?



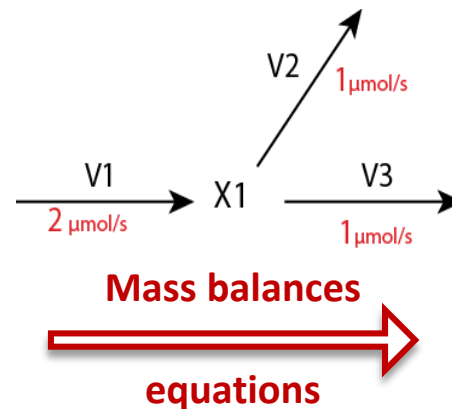
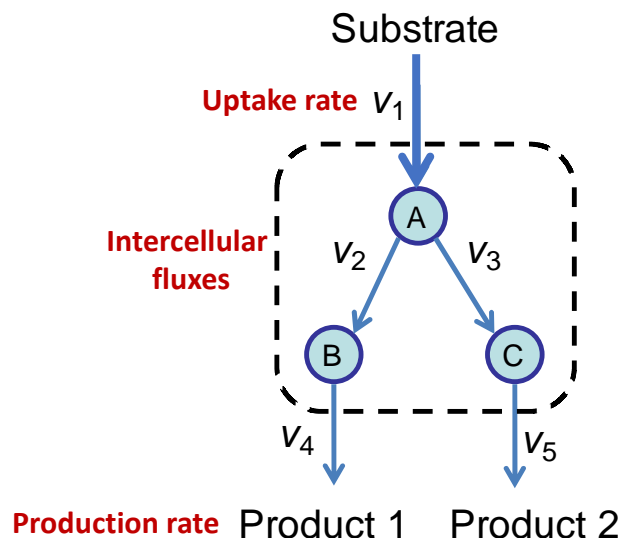
- $v_1$   $R1 : \text{Substrate} \rightarrow A$
- $v_2$   $R2 : A \rightarrow B$
- $v_3$   $R3 : A \rightarrow C$
- $v_4$   $R4 : B \rightarrow \text{product1}$
- $v_5$   $R5 : C \rightarrow \text{product2}$

**Why calculating the intercellular fluxes?**  
**How to calculate the intercellular fluxes?**

# Metabolic Fluxes Analysis

**quantitatively analyze metabolic pathways** (calculating/estimating all the missing fluxes).

- ✓ Simulation of the effect of environmental or genetic changes
- ✓ Identification of important/critical/bottleneck reactions or pathways
- ✓ Understanding/controlling the pathway branching points



$$\frac{d[A]}{dt} = v_1 - v_2 - v_3$$

$$\frac{d[B]}{dt} = v_2 - v_4$$

$$\frac{d[C]}{dt} = v_3 - v_5$$



# (Quasi-)Steady state assumption

- Metabolism does not change with respect to time
  - All intermediates do not accumulate
  - The sum of influxes equals the sum of effluxes



Under the ***steady state assumption***:

$$\frac{d[A]}{dt} = v_1 - v_2 - v_3$$

$$\frac{d[B]}{dt} = v_2 - v_4$$

$$\frac{d[C]}{dt} = v_3 - v_5$$

$$\Rightarrow \frac{d[A]}{dt} = \frac{d[B]}{dt} = \frac{d[C]}{dt} = 0 \Rightarrow$$

$$v_1 - v_2 - v_3 = 0$$

$$v_2 - v_4 = 0$$

$$v_3 - v_5 = 0$$

# (Quasi-)Steady state assumption

The mass balance equations can be described as follows:

$$\begin{aligned}\frac{d[A]}{dt} &= v_1 - v_2 - v_3 = 0 \\ \frac{d[B]}{dt} &= v_2 - v_4 = 0 \\ \frac{d[C]}{dt} &= v_3 - v_5 = 0\end{aligned}$$



**matrix multiplication**

		Reactions				
		R1	R2	R3	R4	R5
Metabolites	Substrate	-1	0	0	0	0
	A	1	-1	-1	0	0
	B	0	1	0	-1	0
	C	0	0	1	0	-1
	Product 1	0	0	0	1	0
	Product 2	0	0	0	0	1

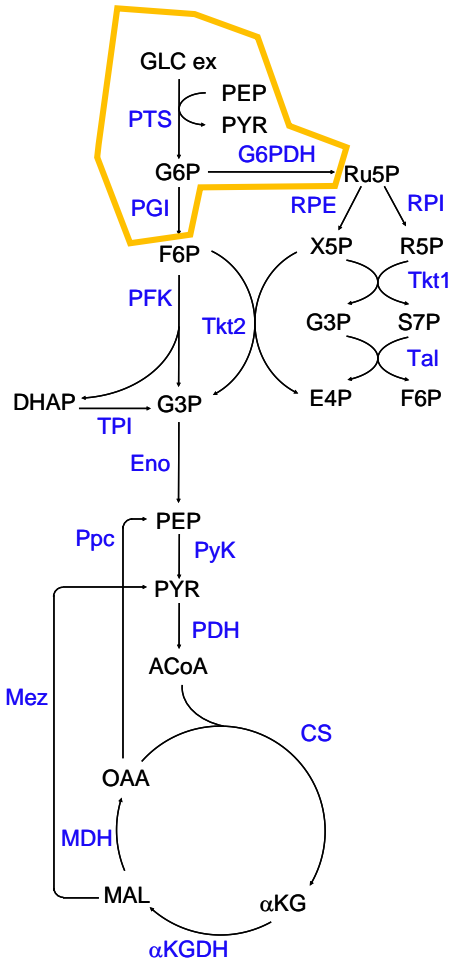
$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}$$

$$= 0 \Rightarrow Sv = 0$$



# Underdetermined Systems

“there are fewer equations than unknowns”



$$v_{PTS} = v_{PGI} + v_{G6PDH}$$

$$10 = v_{PGI} + v_{G6PDH}$$

**2 UNKNOWN  
1 EQUATION**

- 10 & 0
- 1 & 9
- 9 & 1
- 2 & 8
- 3 & 7
- 4 & 6
- 6 & 4
- 5 & 5
- 100 & 110
- 1200 & -1190
- .
- .
- .

**space of hundreds of  
feasible solutions**

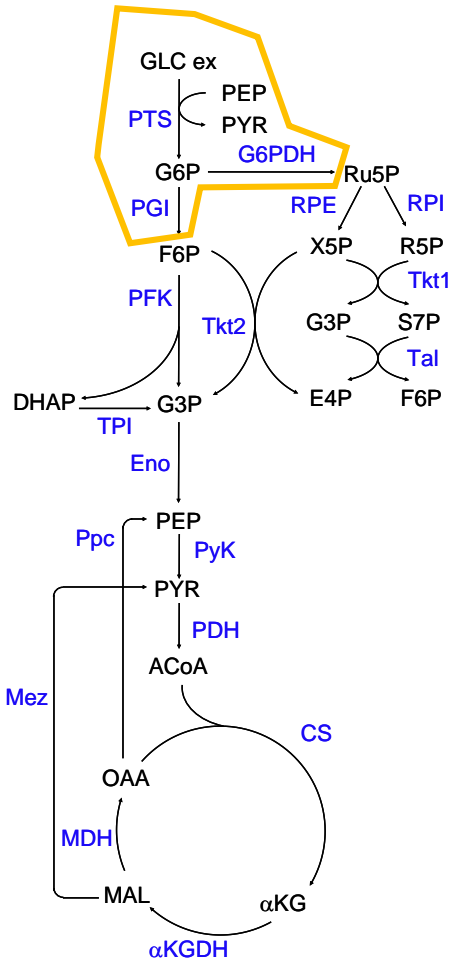
# Constraint-based Modeling

Achieving a certain **objective**  
in a system  
that is shaped by the **defined constraints**.

*There are many different ways from Lausanne to Zurich, but they will all result in reaching Zurich (**objective**) and if I say I should definitely pass through Fribourg & Bern (**defined constraints**) some results will be eliminated.*

# Undetermined Systems

“there are fewer equations than unknowns”



$$V_{PTS} = V_{PGI} + V_{G6PDH}$$

$$10 = V_{PGI} + V_{G6PDH}$$

**2 UNKNOWN-1 EQUATION**

Transcriptomics

~~10 & 0~~

~~1 & 9~~

~~9 & 1~~

2 & 8

3 & 7

~~-20 & 30~~

6 & 4

5 & 5

~~-100 & 110~~

~~1200 & -1190~~

.

.

.

Thermodynamics

Fluxomics

**space of hundreds of feasible solutions**

# Flux Balance Analysis

from math to growth

# Back to metabolic models ....

Equality constrains:

Stoichiometry:

$$Sv = 0$$

ATP	-1	0	-1	0	0	0	1	0	0	1
GLC	-1	0	0	0	0	0	0	0	0	0
ADP	1	0	1	0	0	0	-1	0	0	-1
G6P	1	-1	0	0	0	0	0	0	0	0
H	1	0	1	0	0	1	0	0	0	-1
F6P	0	1	-1	0	0	0	0	0	0	0
FDP	0	0	1	-1	0	0	0	0	0	0
DHAP	0	0	0	1	-1	0	0	0	0	0
G3P	0	0	0	1	1	-1	0	0	0	0
NAD	0	0	0	0	0	-1	0	0	0	0
PI	0	0	0	0	0	-1	0	0	0	0
13DPG	0	0	0	0	0	1	-1	0	0	0
NADH	0	0	0	0	0	1	0	0	0	0
3PG	0	0	0	0	0	0	1	-1	0	0
2PG	0	0	0	0	0	0	0	1	-1	0
PEP	0	0	0	0	0	0	0	0	1	-1
H <sub>2</sub> O	0	0	0	0	0	0	0	0	1	0
PYR	0	0	0	0	0	0	0	0	0	1
	HEX1	PGI	PFK	FBA	TPI	GAPD	PGK	PGM	ENO	PYK

Inequality constraints:

Which substrates are available?

Which of them are less abundant?

What can be secreted?

# Constraints of metabolism

Equality constrains:

Stoichiometry:

$$Sv = 0$$

ATP	-1	0	-1	0	0	0	1	0	0	1
GLC	-1	0	0	0	0	0	0	0	0	0
ADP	1	0	1	0	0	0	-1	0	0	-1
G6P	1	-1	0	0	0	0	0	0	0	0
H <sub>2</sub>	1	0	1	0	0	1	0	0	0	-1
F6P	0	1	-1	0	0	0	0	0	0	0
FDP	0	0	1	-1	0	0	0	0	0	0
DHAP	0	0	0	1	-1	0	0	0	0	0
G3P	0	0	0	1	1	-1	0	0	0	0
NAD	0	0	0	0	0	-1	0	0	0	0
PI	0	0	0	0	0	-1	0	0	0	0
13DPG	0	0	0	0	0	1	-1	0	0	0
NADH	0	0	0	0	0	1	0	0	0	0
3PG	0	0	0	0	0	0	1	-1	0	0
2PG	0	0	0	0	0	0	0	1	-1	0
PEP	0	0	0	0	0	0	0	1	-1	0
H <sub>2</sub> O	0	0	0	0	0	0	0	0	1	0
PYR	0	0	0	0	0	0	0	0	0	1
	HEX1	PGI	PFK	FBA	TPI	GAPD	PGK	PGM	ENO	PYK

Inequality constraints:

Uptakes:

$$-10 \leq v_{EX\_glu\_e} \leq 0$$

$$-20 \leq v_{EX\_O2\_e} \leq 0$$

Secretions:

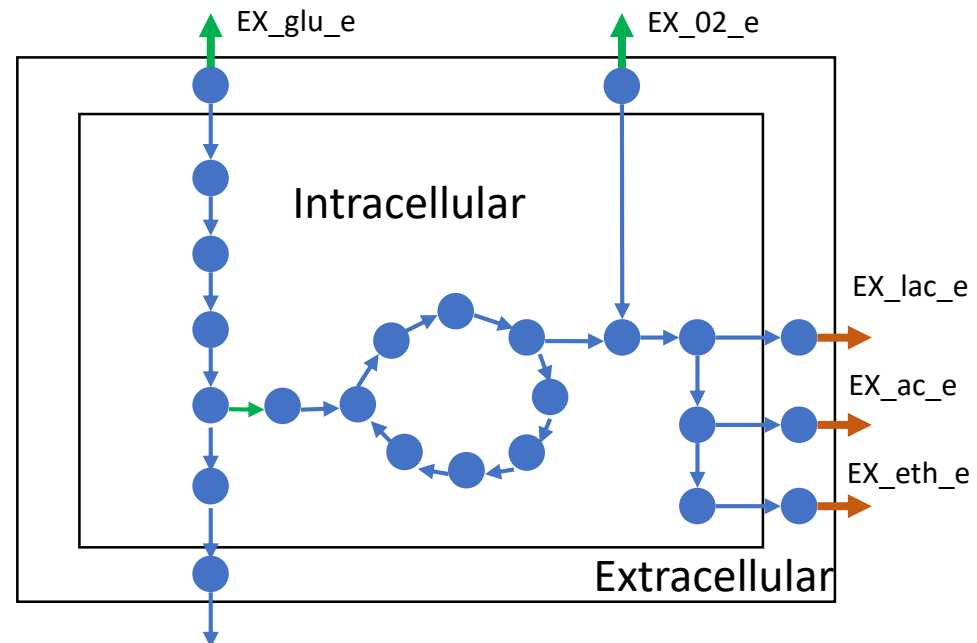
$$0 \leq v_{EX\_lac\_e} \leq 1000$$

$$0 \leq v_{EX\_ac\_e} \leq 1000$$

$$0 \leq v_{EX\_eth\_e} \leq 1000$$

Internal fluxes:

$$10 \leq v_{PYK} \leq 20$$



# Constraints of metabolism

Equality constrains:

Stoichiometry:  $Sv = 0$

ATP	-1	0	-1	0	0	0	1	0	0	1
GLC	-1	0	0	0	0	0	0	0	0	0
ADP	1	0	1	0	0	0	-1	0	0	-1
G6P	1	-1	0	0	0	0	0	0	0	0
H	1	0	1	0	0	1	0	0	0	-1
F6P	0	1	-1	0	0	0	0	0	0	0
FDP	0	0	1	-1	0	0	0	0	0	0
DHAP	0	0	0	1	-1	0	0	0	0	0
G3P	0	0	0	1	1	-1	0	0	0	0
NAD	0	0	0	0	0	-1	0	0	0	0
PI	0	0	0	0	0	-1	0	0	0	0
13DPG	0	0	0	0	0	1	-1	0	0	0
NADH	0	0	0	0	0	1	0	0	0	0
3PG	0	0	0	0	0	0	1	-1	0	0
2PG	0	0	0	0	0	0	0	1	-1	0
PEP	0	0	0	0	0	0	0	0	1	-1
H <sub>2</sub> O	0	0	0	0	0	0	0	0	1	0
PYR	0	0	0	0	0	0	0	0	0	1
	HEX1	PGI	PFK	FBA	TPI	GAPD	PGK	PGM	ENO	PYK

Inequality constraints:

Uptakes:  $-10 \leq v_{EX\_glu\_e} \leq 0$

$-20 \leq v_{EX\_O2\_e} \leq 0$

Secretions:  $0 \leq v_{EX\_lac\_e} \leq 1000$

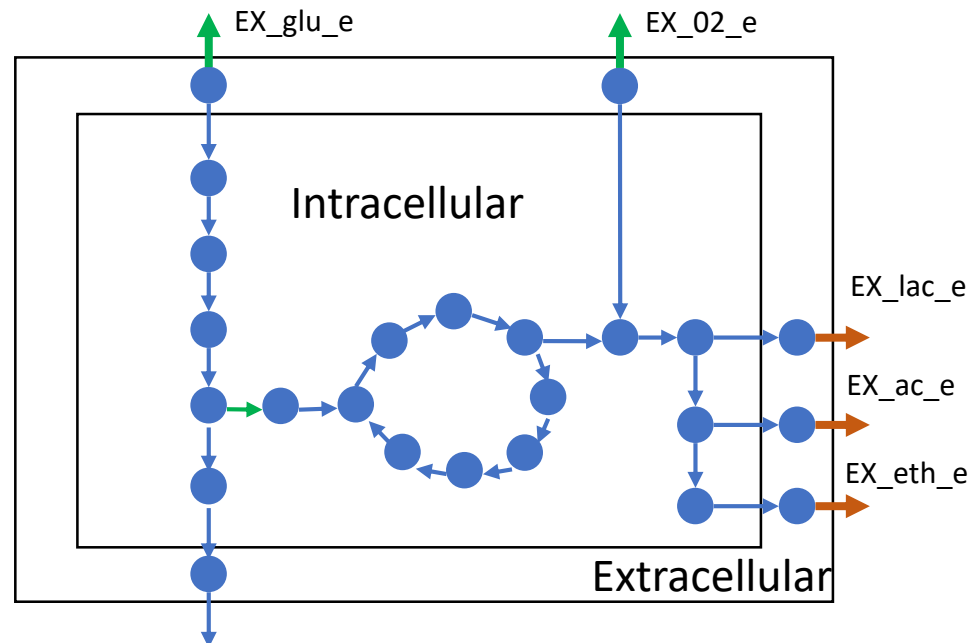
$0 \leq v_{EX\_ac\_e} \leq 1000$

$0 \leq v_{EX\_eth\_e} \leq 1000$

Internal fluxes:

$10 \leq v_{PYK} \leq 20$

If available from data....



# Does metabolism have an “Objective” ?

Stoichiometry:

$$Sv = 0$$

Uptakes:

$$-10 \leq v_{EX\_glu\_e} \leq 0$$

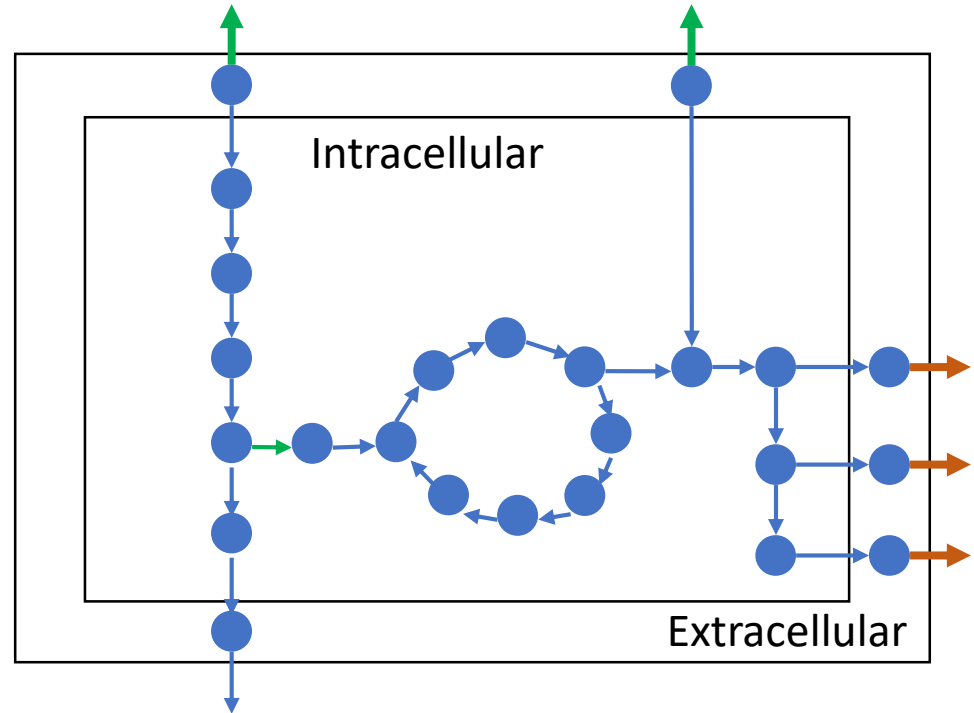
$$-20 \leq v_{EX\_O2\_e} \leq 0$$

Secretions:

$$0 \leq v_{EX\_lac\_e} \leq 1000$$

$$0 \leq v_{EX\_ac\_e} \leq 1000$$

$$0 \leq v_{EX\_eth\_e} \leq 1000$$



What is the objective function?

# Does metabolism have an “Objective” ?

Stoichiometry:

$$Sv = 0$$

Uptakes:

$$-10 \leq v_{EX\_glu\_e} \leq 0$$

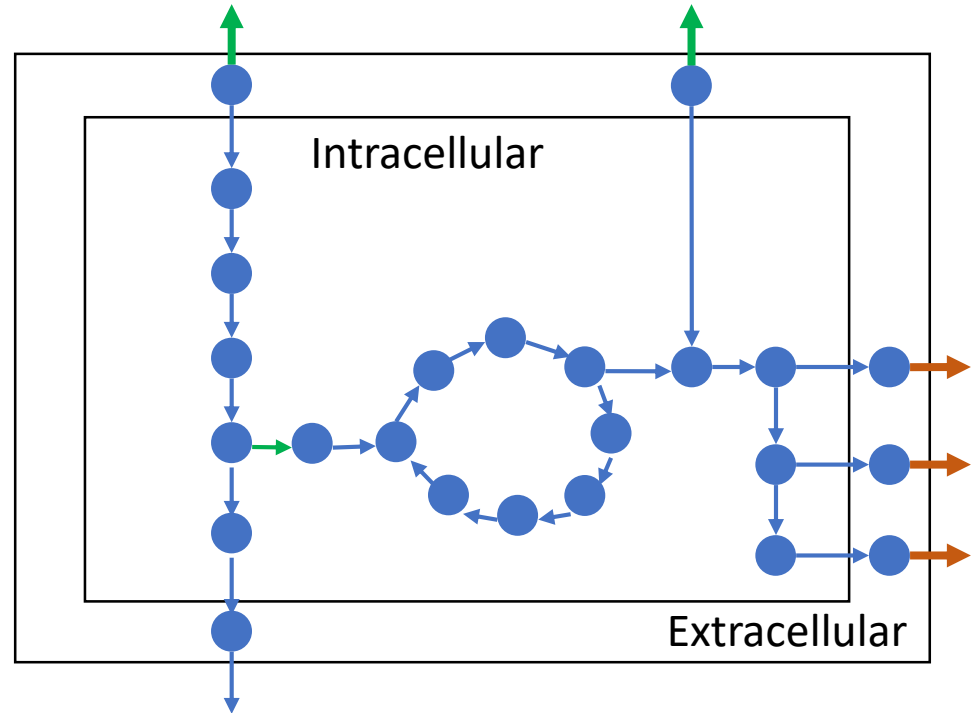
$$-20 \leq v_{EX\_O2\_e} \leq 0$$

Secretions:

$$0 \leq v_{EX\_lac\_e} \leq 1000$$

$$0 \leq v_{EX\_ac\_e} \leq 1000$$

$$0 \leq v_{EX\_eth\_e} \leq 1000$$



## What is the objective function?

Evolutionary objective: Utilize **minimal** necessary **resources** for **maximum “growth”**

Does metabolism have an “*Objective*” ?

**What is growth?**

# Does metabolism have an “*Objective*” ?

## What is growth?

... doubling time

... maximum theoretical growth rate given  
a maximum theoretical uptake rate

... specific growth rate, yield

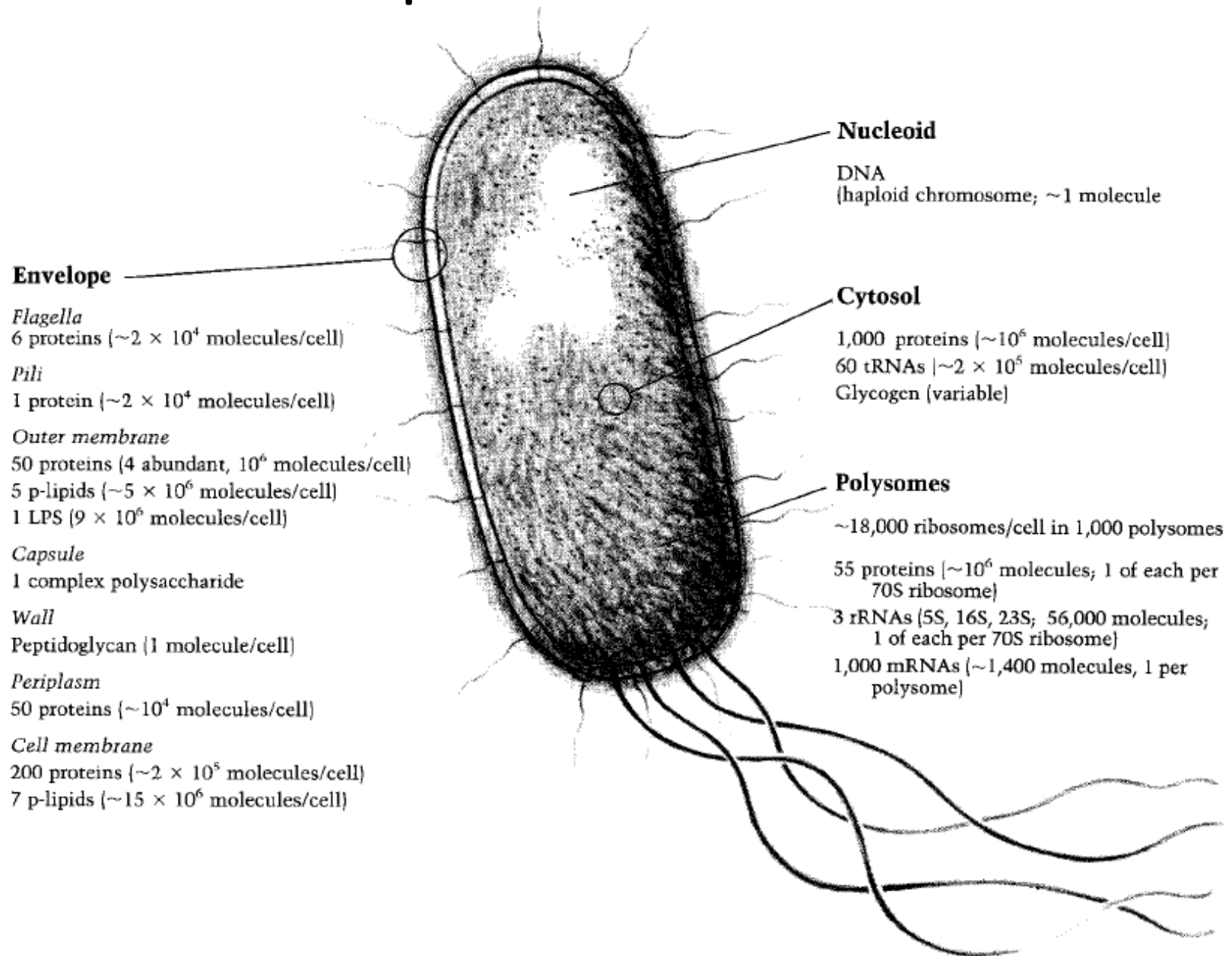
Phase diagram ... slope of log growth

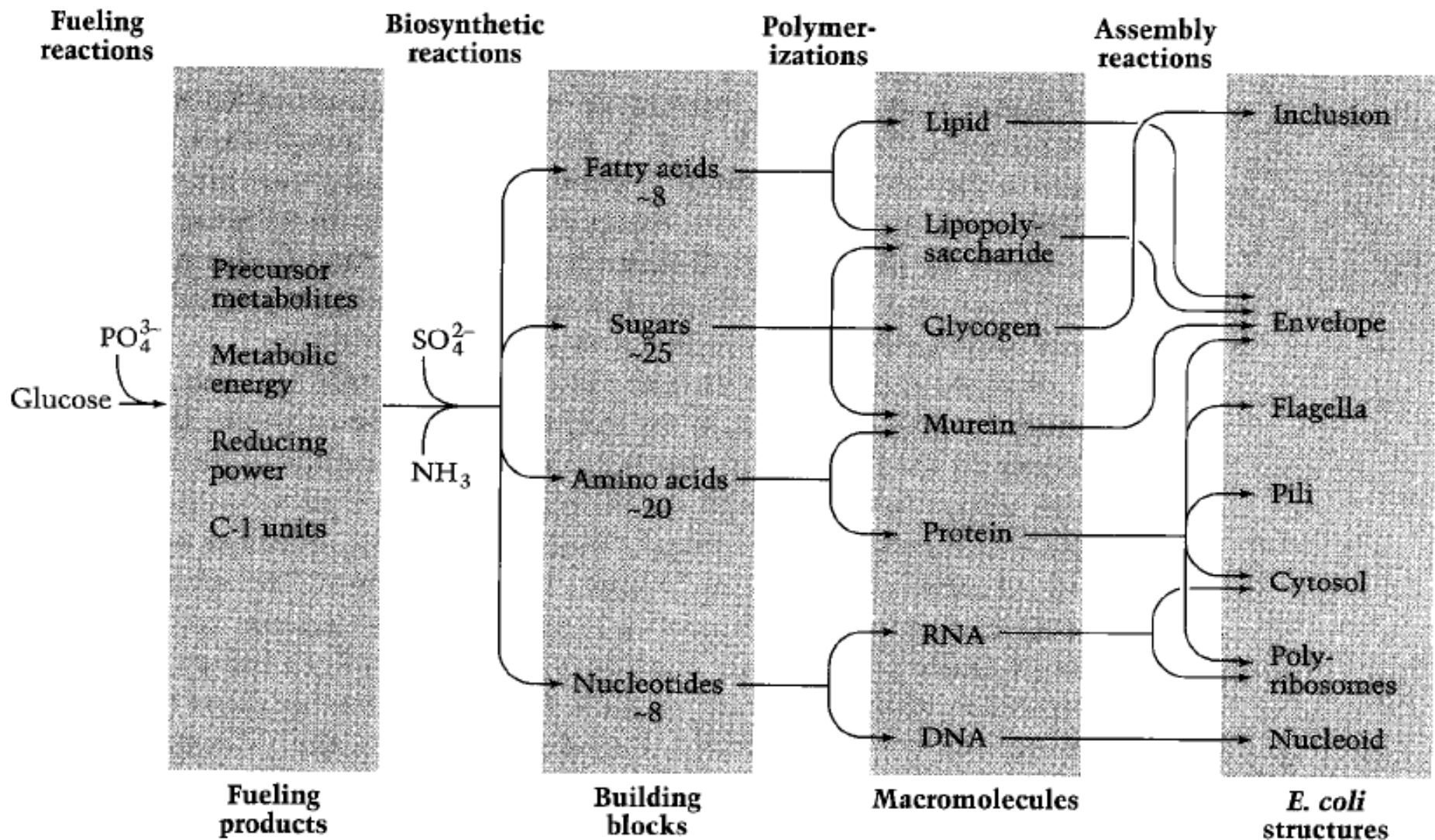
*Is maximization of growth  
an objective of the cell?*

*(Darwin...)*

**How to define growth in the model?**

# Biomass composition





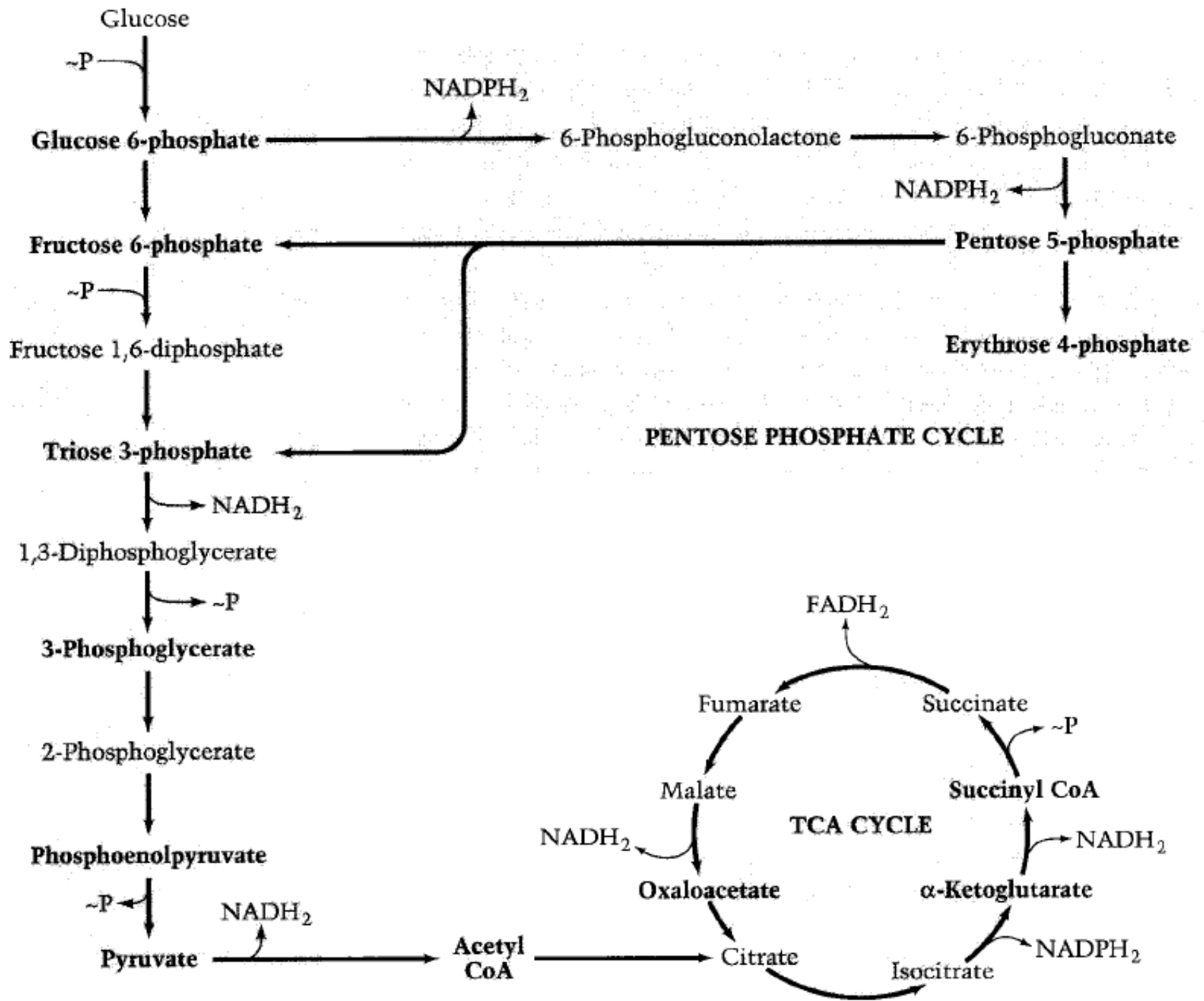
**Figure 1**

**Overview of metabolism leading to the chemical synthesis from glucose of a chemoheterotroph like *E. coli*.**

*Table 3.* Costs of biosynthesis of cellular components from the precursor metabolites

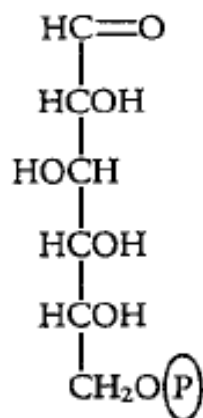
Cellular component	Energy cost <sup>a</sup> ( $\mu$ mole $\sim$ P/g cells)	Reducing power cost ( $\mu$ mole NADPH/g cells)
Protein	7,287	11,523
RNA	6,540	427
DNA	1,090	200
Lipid	2,578	5,270
LPS	470	564
Murein	248	193
Glycogen	154	0
1-Carbon	0	48
Polyamines	<u>118</u>	<u>0</u>
Total	18,485	18,225

<sup>a</sup>Each nucleoside triphosphate is assumed to be made by consecutive reactions with ATP that consume 3  $\sim$ P per NTP produced. Formation of sugar-nucleotide derivatives are assumed to occur by direct reaction with the appropriate nucleoside triphosphate.

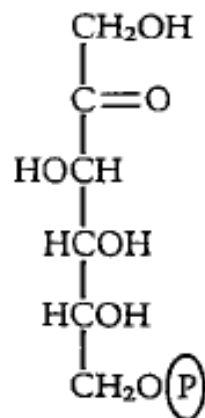


EMP PATHWAY

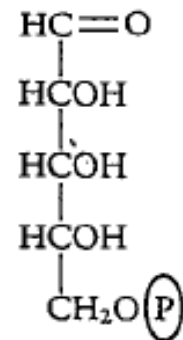
Figure 4



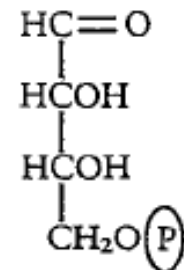
**Glucose 6-phosphate**



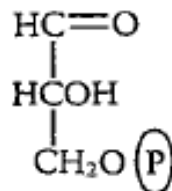
**Fructose 6-phosphate**



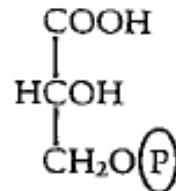
**Ribose 5-phosphate**



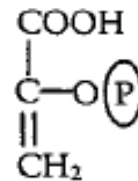
**Erythrose 4-phosphate**



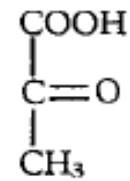
**Triose phosphate**



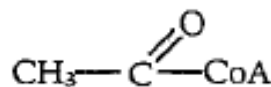
**3-Phosphoglycerate**



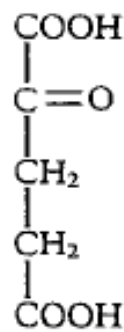
**Phosphoenolpyruvate**



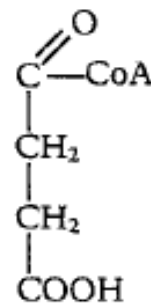
**Pyruvate**



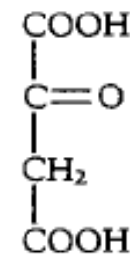
**Acetyl CoA**



**$\alpha$ -Ketoglutarate**



**Succinyl CoA**



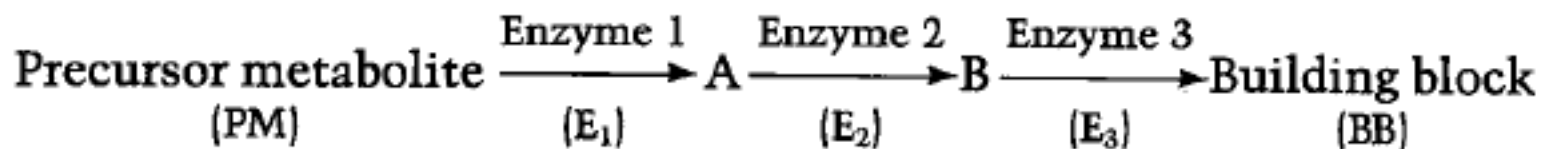
**Oxaloacetate**

**Figure 1**

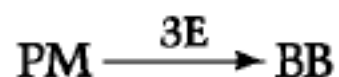
**Structures of the 12 precursor metabolites.**

## Biosynthetic pathways

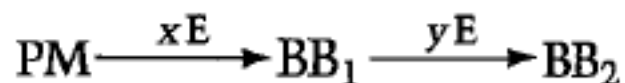
Biosynthetic pathways differ markedly in complexity—some are linear, others branch or are interconnected. A simple pathway, consisting of three sequential enzymatic reactions, might be represented as



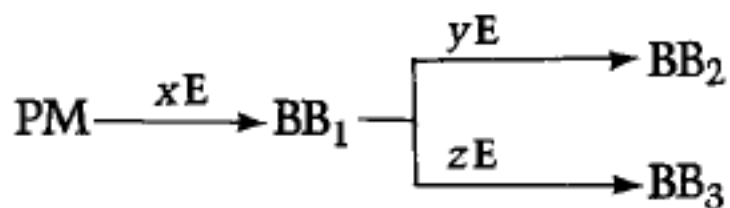
where A and B are intermediate products in the pathway. Recognizing



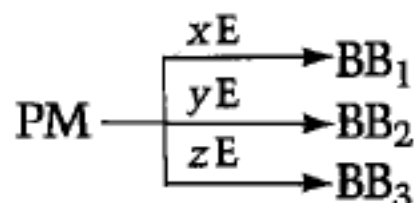
Some pathways produce a building block that, in turn, is converted by a second pathway into another building block:



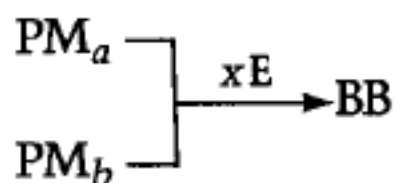
where  $x$  and  $y$  represent the number of enzymes in the two pathways. In some cases the pathway is branched:



Most of the 12 precursor metabolites actually serve as the starting point for several pathways:



Finally, in many cases more than one precursor molecule is involved in the biosynthesis of a building block:



Branching and interlocking of this sort are common among biosynthetic pathways. Building blocks that are produced from a common precursor are called a FAMILY. The ASPARTATE FAMILY, for example, consists of seven amino acids (asparagine, aspartate, diaminopimelate, isoleucine, lysine, methionine, and threonine) that are synthesized from the common precursor metabolite oxaloacetate (Figure 2).

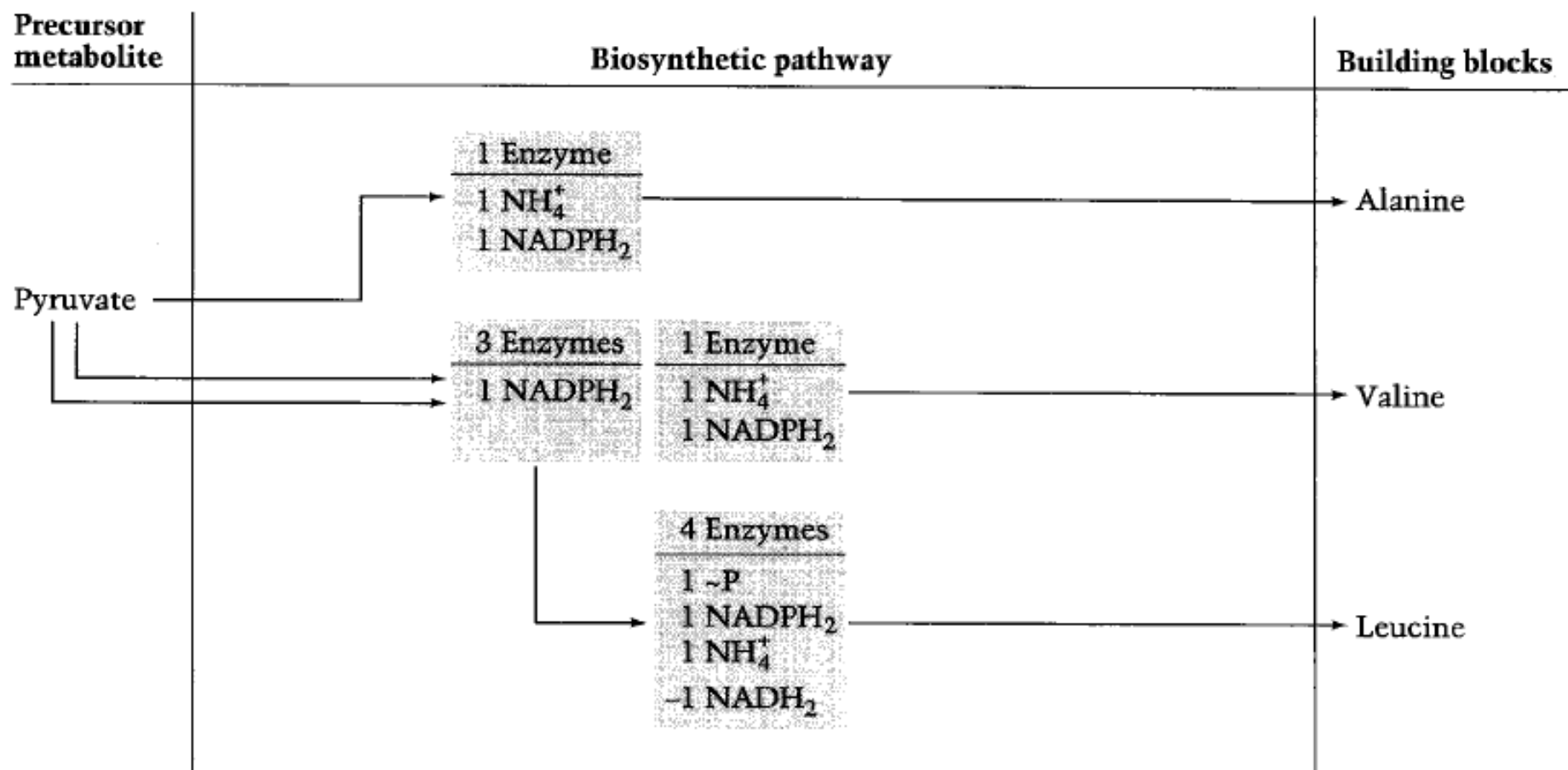
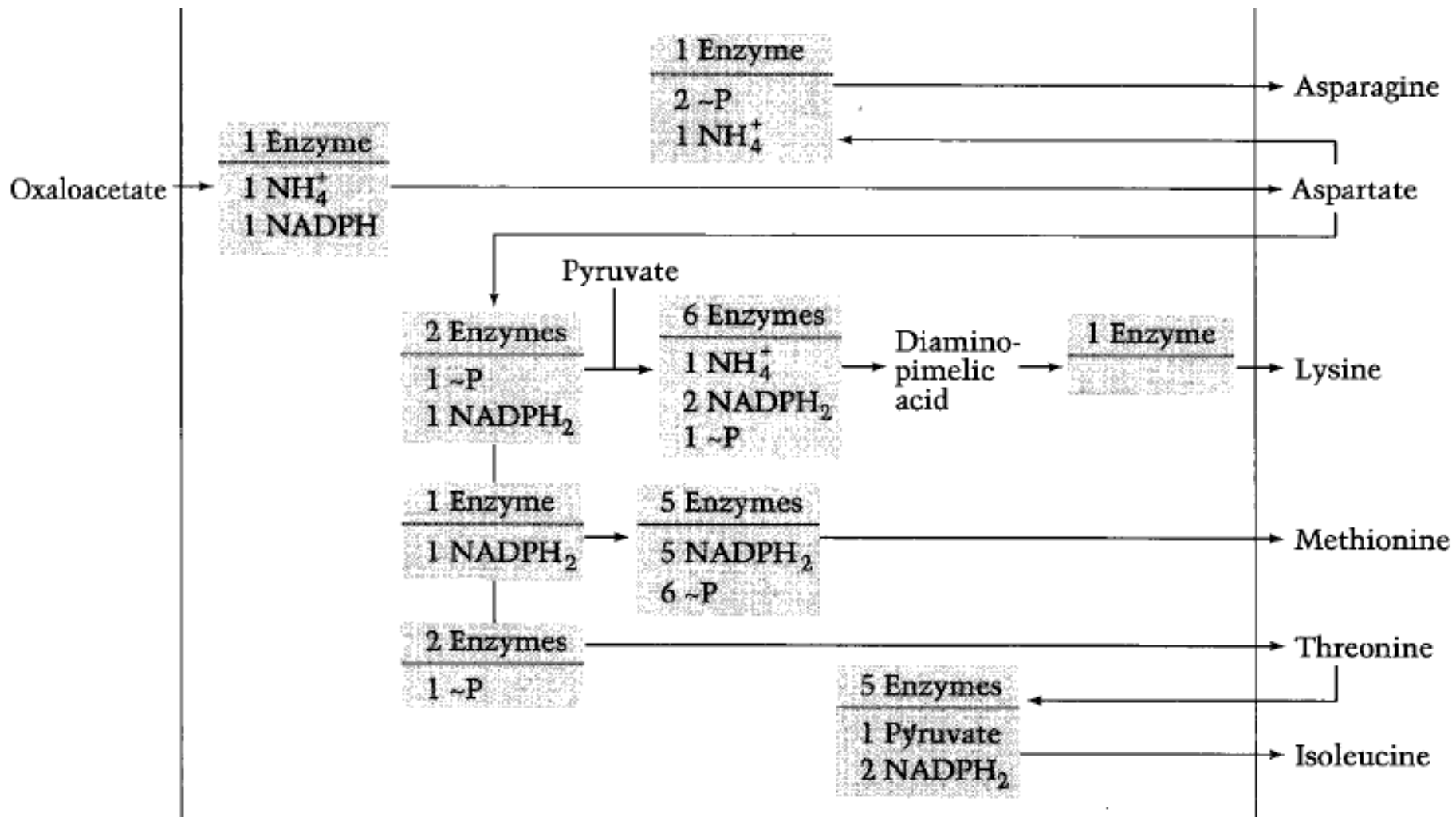
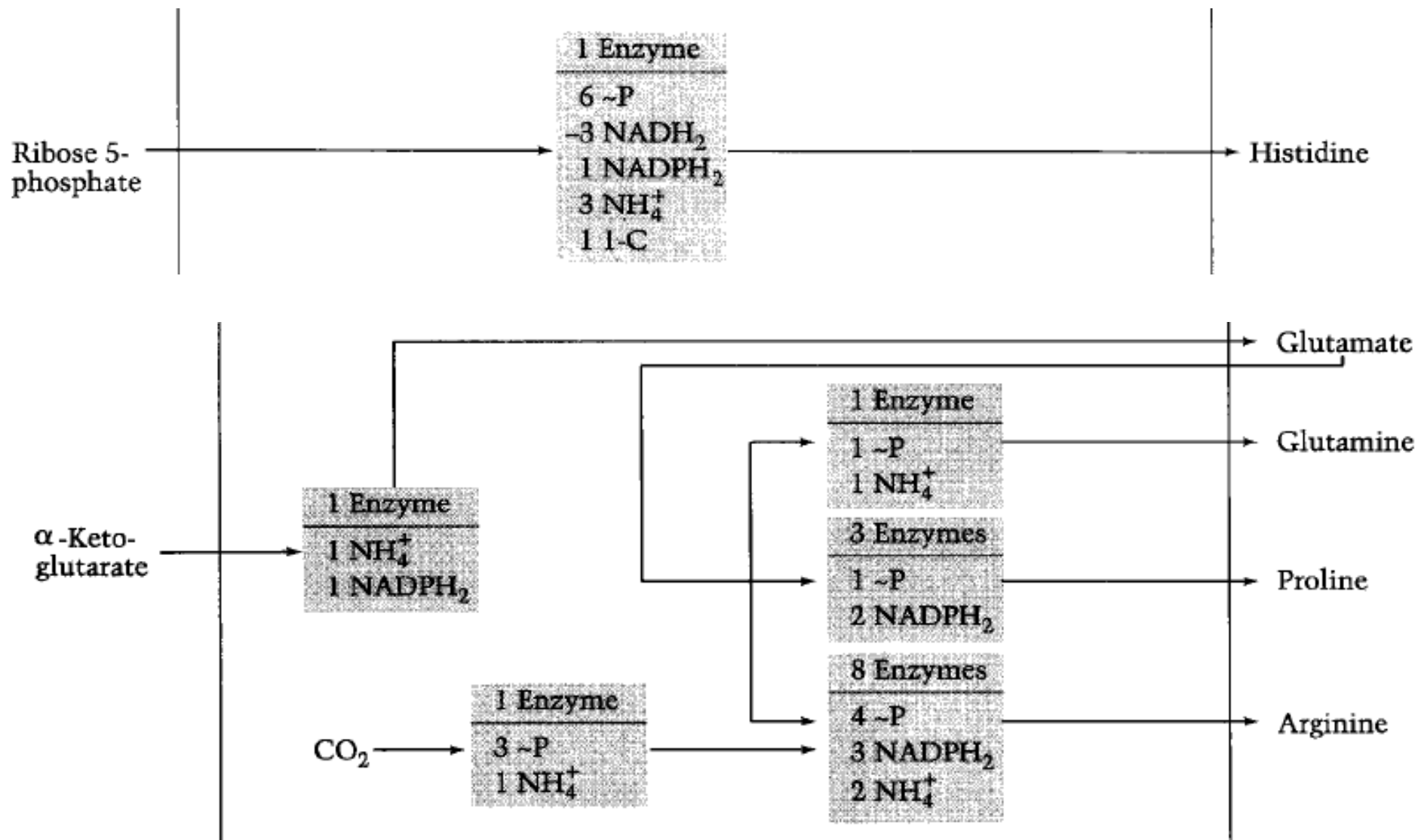
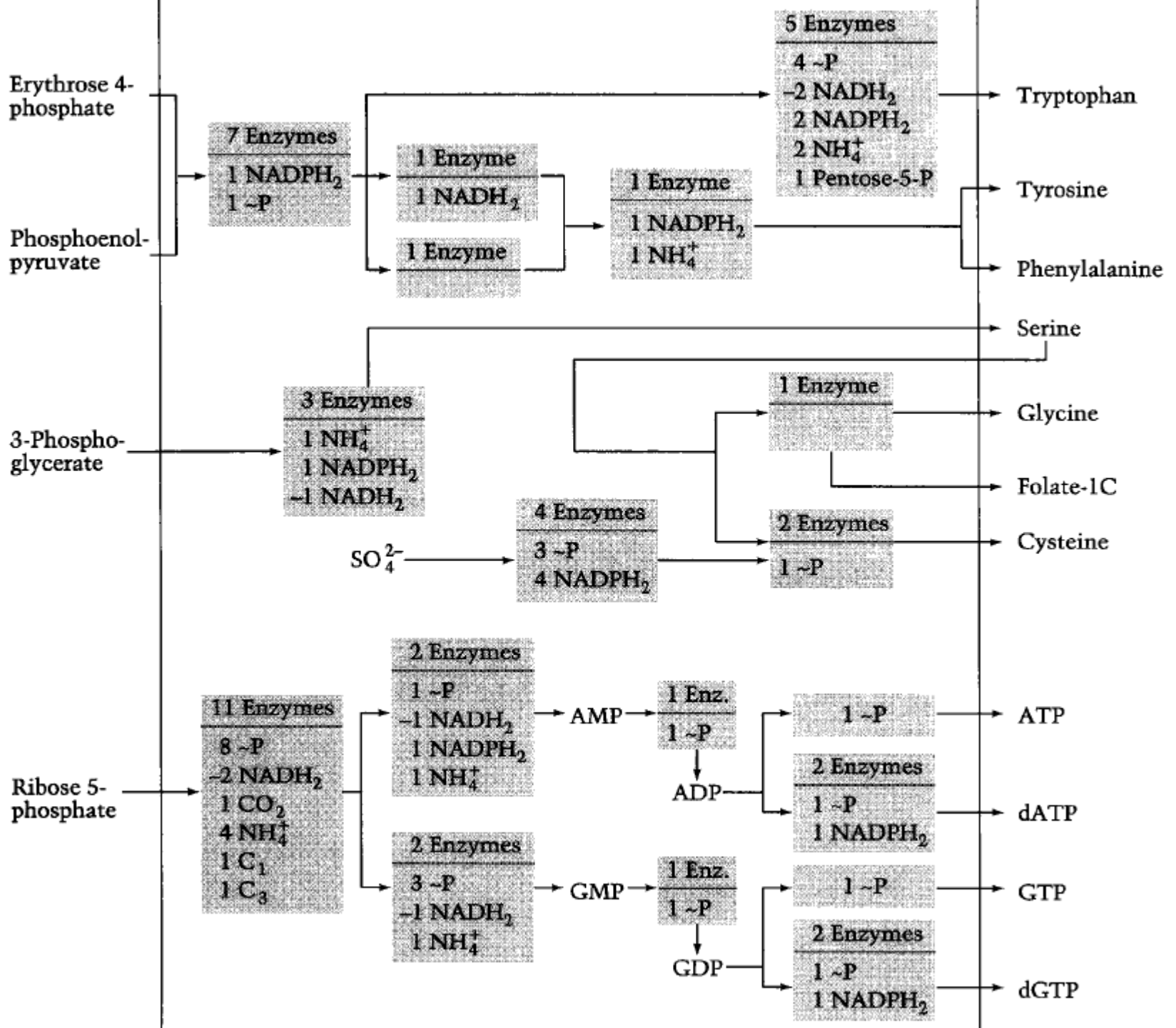


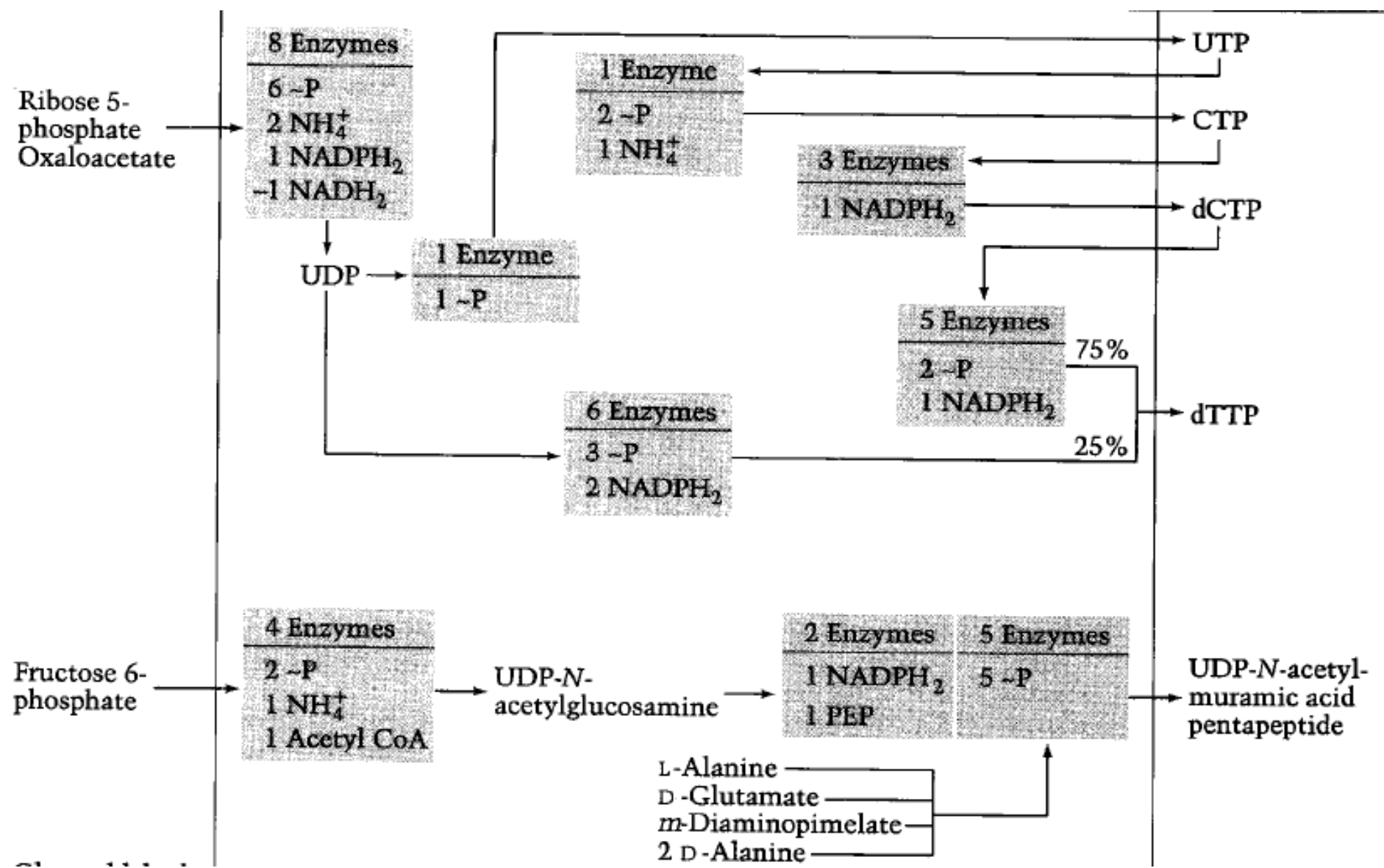
Table 1. Building blocks needed to produce 1 g of *E. coli* protoplasm

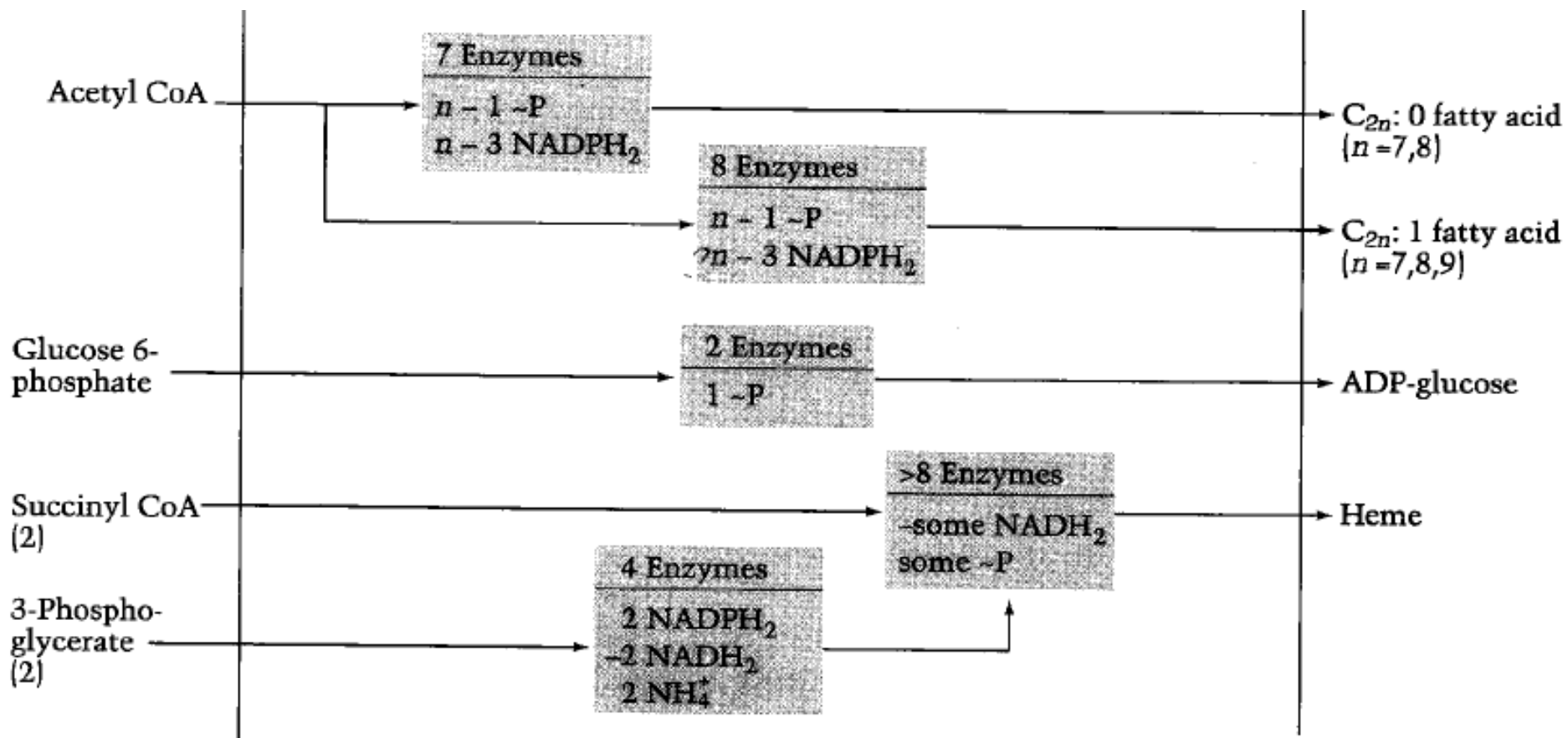
Building block	Amount present in <i>E. coli</i> B/r ( $\mu\text{mol/g}$ dried cells)	Cost of making 1 $\mu\text{mol}$ of each of these building blocks ( $\mu\text{mol}/\mu\text{mol}$ )						
		Metabolites <sup>a</sup>	ATP	NADH	NADPH	1-C	NH <sub>4</sub> <sup>+</sup>	S
<b>Protein amino acids</b>								
Alanine	488	1 pyr	0	0	1	0	1	0
Arginine	281	1 $\alpha\text{kg}$	7	-1	4	0	4	0
Asparagine	229	1 oaa	3	0	1	0	2	0
Aspartate	229	1 oaa	0	0	1	0	1	0
Cysteine	87	1 pga	4	-1	5	0	1	1
Glutamate	250	1 $\alpha\text{kg}$	0	0	1	0	1	0
Glutamine	250	1 $\alpha\text{kg}$	1	0	1	0	2	0
Glycine	582	1 pga	0	-1	1	-1	1	0
Histidine	90	1 penP	6	-3	1	1	3	0
Isoleucine	276	1 oaa, 1 pyr	2	0	5	0	1	0
Leucine	428	2 pyr, 1 acCoA	0	-1	2	0	1	0
Lysine	326	1 oaa, 1 pyr	2	0	4	0	2	0
Methionine	146	1 oaa	7	0	8	1	1	1
Phenylalanine	176	1 eryP, 2 pep	1	0	2	0	1	0
Proline	210	1 $\alpha\text{kg}$	1	0	3	0	1	0
Serine	205	1 pga	0	-1	1	0	1	0
Threonine	241	1 oaa	2	0	3	0	1	0
Tryptophan	54	1 penP, 1 eryP, 1 pep	5	-2	3	0	2	0
Tyrosine	131	1 eryP, 2 pep	1	-1	2	0	1	0
Valine	402	2 pyr	0	0	2	0	1	0











**RNA nucleotides**

ATP	165	1 penP, 1 pga	11	-3	1	1	5	0
GTP	203	1 penP, 1 pga	13	-3	0	1	5	0
CTP	126	1 penP, 1 oaa	9	0	1	0	3	0
UTP	136	1 penP, 1 oaa	7	0	1	0	2	0

**DNA nucleotides**

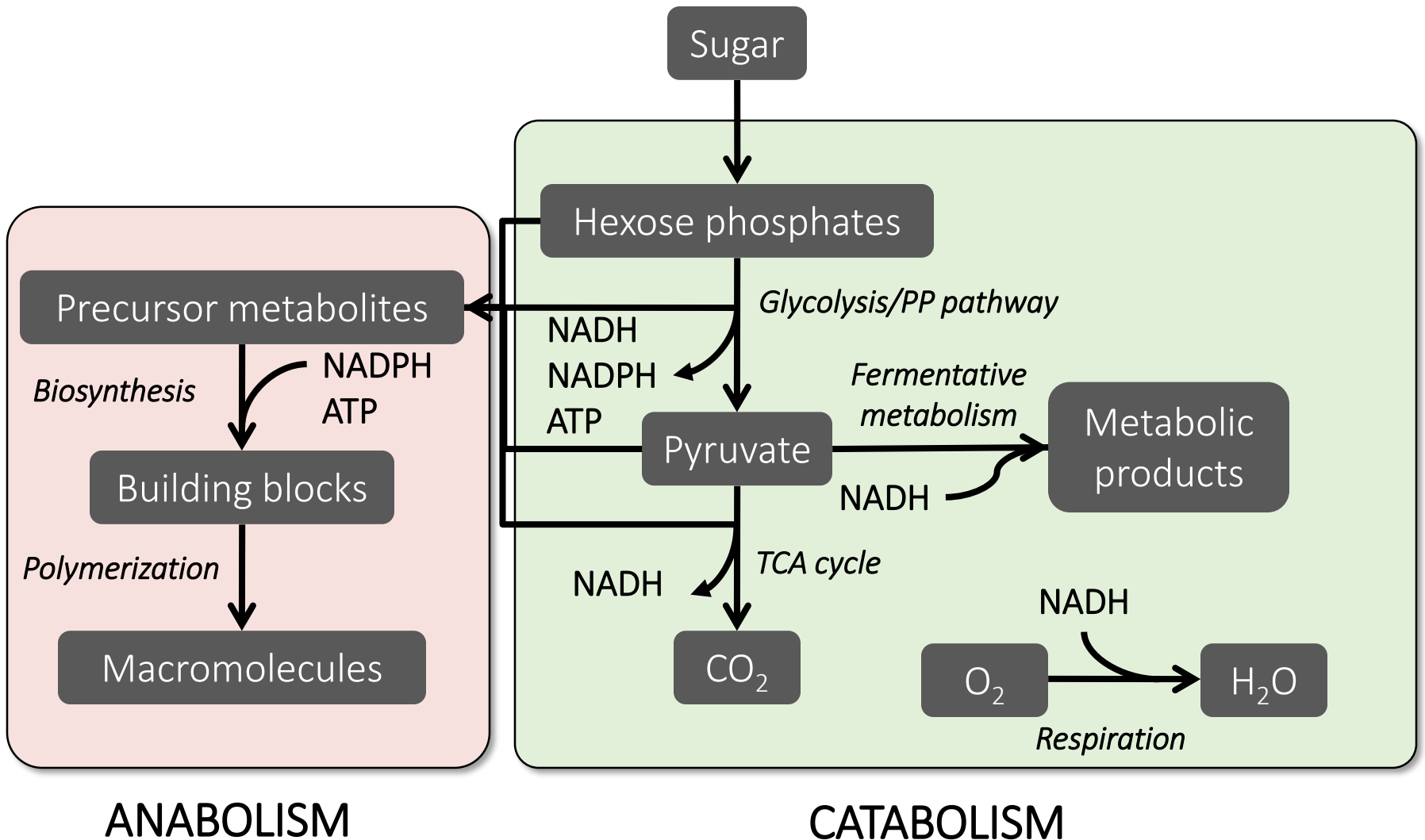
dATP	24.7	1 penP, 1 pga	11	-3	2	1	5	0
dGTP	25.4	1 penP, 1 pga	13	-3	1	1	5	0
dCTP	25.4	1 penP, 1 oaa	9	0	2	0	3	0
dTTP	24.7	1 penP, 1 oaa	10.5	0	3	1	2	0

**Lipid components**

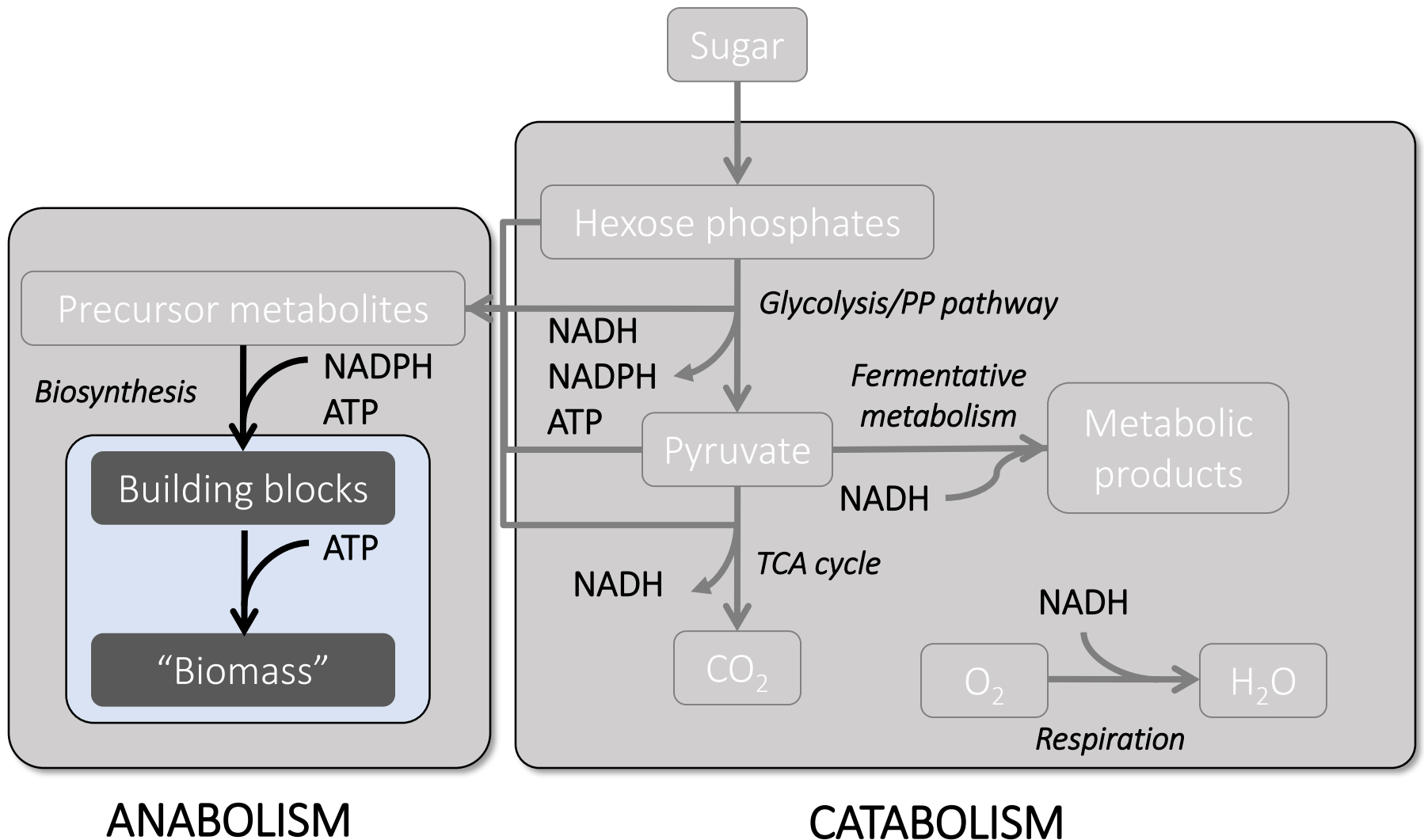
Glycerol phosphate	129	1 triosP	0	0	1	0	0	0
Serine	129	1 pga	0	-1	1	0	1	0
C <sub>16:0</sub> fatty acid (43%)		8 acCoA	7	0	14	0	0	0
C <sub>16:1</sub> fatty acid (33%)		8 acCoA	7	0	13	0	0	0
C <sub>18:1</sub> fatty acid (24%)		9 acCoA	8	0	15	0	0	0
Average fatty acid	258	8.2 acCoA	7.2	0	14	0	0	0

Building block	Amount present in <i>E. coli</i> B/r ( $\mu\text{mol/g}$ dried cells)	Cost of making 1 $\mu\text{mol}$ of each of these building blocks ( $\mu\text{mol}/\mu\text{mol}$ )						
		Metabolites <sup>a</sup>	ATP	NADH	NADPH	1-C	NH <sub>4</sub> <sup>+</sup>	S
<b>LPS components</b>								
UDP-glucose	15.7	1 gluP	1	0	0	0	0	0
(CDP) ethanolamine	23.5	1 pga	3	-1	1	0	1	0
OH-myristic acid	23.5	7 acCoA	6	0	11	0	0	0
C <sub>14:0</sub> fatty acid	23.5	7 acCoA	6	0	12	0	0	0
(CMP) KDO	23.5	1 penP, 1 pep	2	0	0	0	0	0
(NDP) heptose	23.5	1.5 gluP	1	0	-4	0	0	0
(TDP) glucosamine	15.7	1 fruP	2	0	0	0	1	0
<b>Peptidoglycan monomers</b>								
UDP-N-acetylglucosamine	27.6	1 fruP, 1 acCoA	3	0	0	0	1	0
UDP-N-acetylmuramic acid	27.6	1 fruP, 1 pep, 1 acCoA	4	0	1	0	1	0
Alanine	55.2	1 pyr	0	0	1	0	1	0
Diaminopimelate	27.6	1 oaa, 1 pyr	2	0	3	0	2	0
Glutamate	27.6	1 $\alpha\text{kg}$	0	0	1	0	1	0
<b>Glycogen monomers</b>								
Glucose	154	1 gluP	1	0	0	0	0	0
<b>1-Carbon requirement</b>								
Serine	48.5	1 pga	0	-1	1	0	0	0
<b>Polyamines</b>								
Ornithine equivalents	59.3	1 $\alpha\text{kg}$	2	0	3	0	2	0

# The biomass reactions



# The biomass reactions



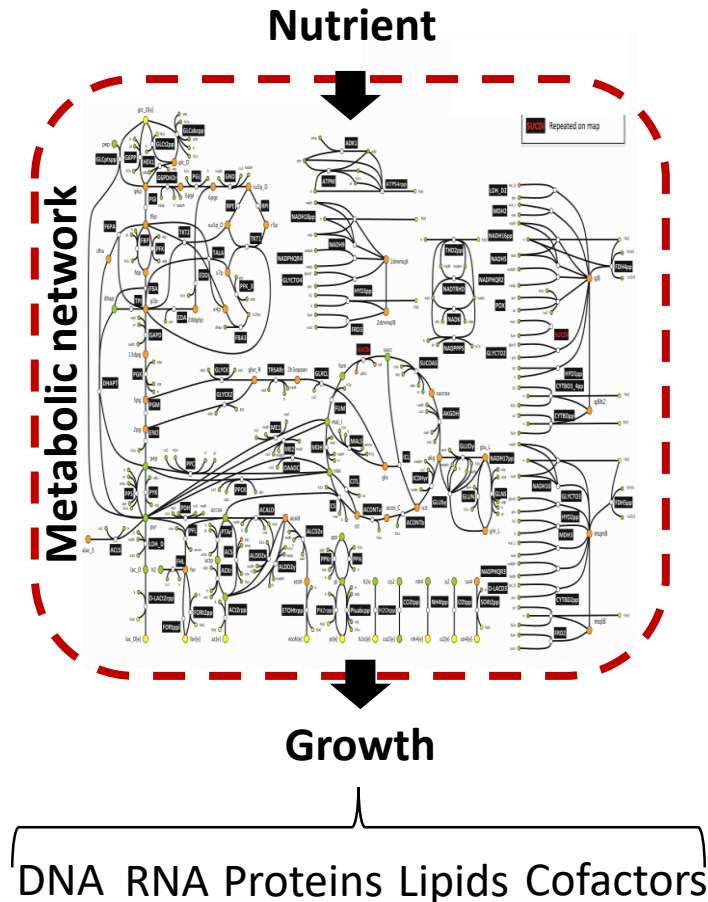
# The biomass reactions

## Biomass building blocks

### Macromolecules

1g Biomass	DNA	6.7 %	AMP	38.7
	RNA	5.9 %	UMP	38.7
	Protein	48.0 %	GMP	11.3
	Lipids	14.1 %	CMP	11.3
	Carbohydrates a.o.	25.3 %		
				etc.

# The biomass reactions



- Organism-specific models
- Start from sequenced genome
- Correlate genome with molecular physiology



Malaria Parasite Metabolic Pathways

# Defining the growth problem

Equality constrains:

$$Sv = 0$$

Inequality constrains:

$$-10 \leq v_{EX\_glu\_e} \leq 0$$

$$-20 \leq v_{EX\_O2\_e} \leq 0$$

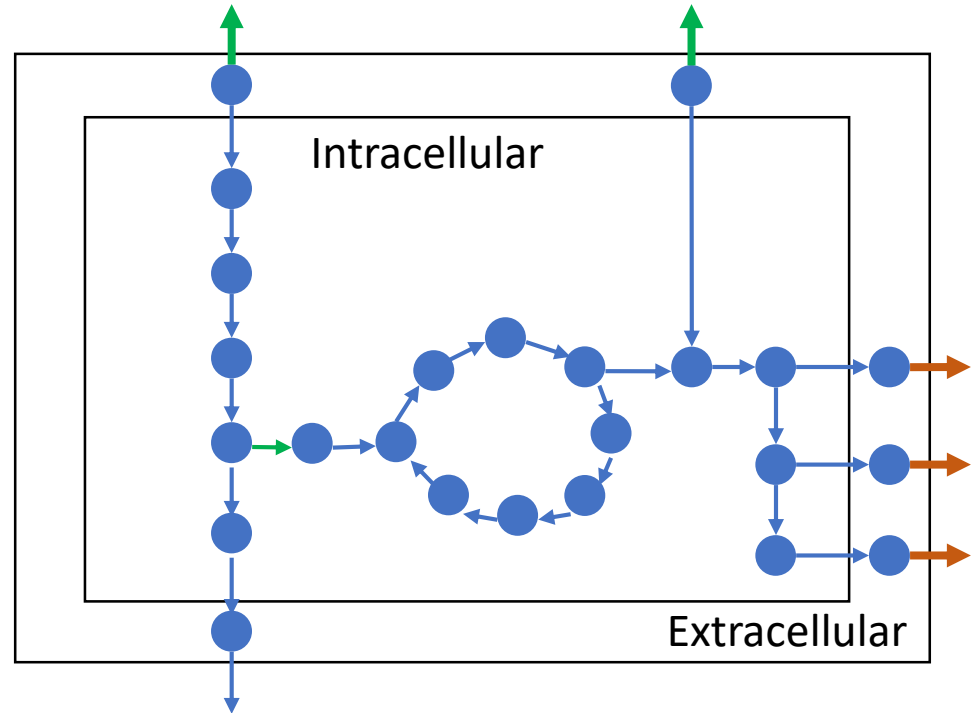
$$0 \leq v_{EX\_lac\_e} \leq 1000$$

$$0 \leq v_{EX\_ac\_e} \leq 1000$$

$$0 \leq v_{EX\_eth\_e} \leq 1000$$

Objective function:

$$\max(\text{biomass reaction}):$$



# Linear programming

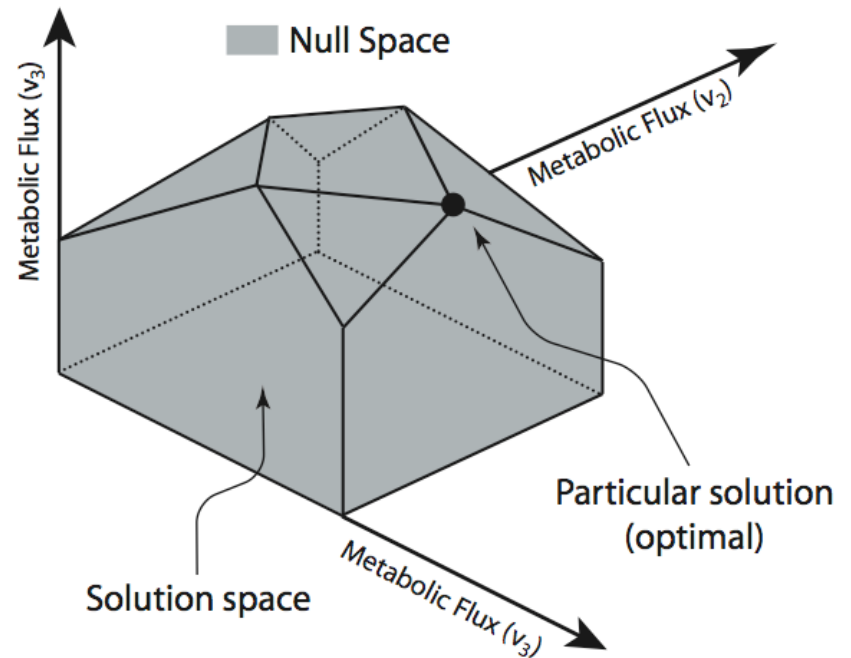
## 1. Forming the Solution Space

$$Sv = 0 \quad , \quad \begin{array}{l} 0 \leq v_i \leq v_{i,m} \\ \min \leq b_i \leq \max \end{array} \quad \begin{array}{l} \text{internal reactions} \\ \text{inputs \& outputs} \end{array}$$

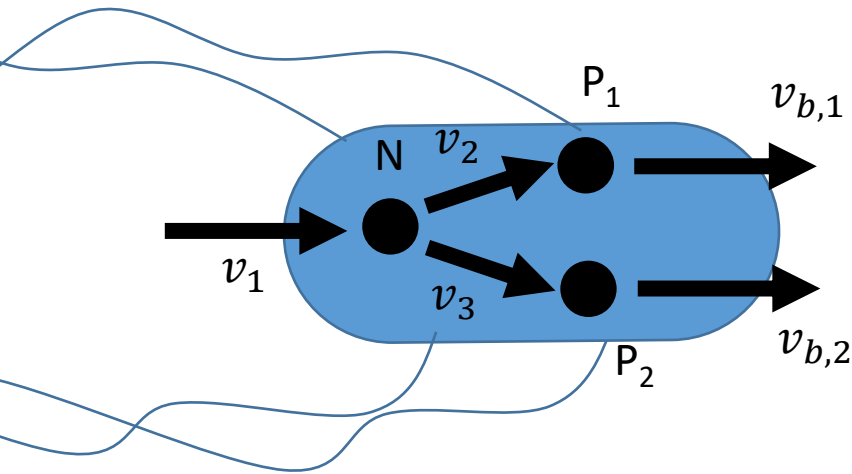
## 2. Objective Function

$$Z = \sum_{i=1}^n c_i v_i$$

Originally from **economics** to **optimize** production processes subject to **linear constraints**.



# How Does LP work: 2D example



Biomass building block  $b_1$  and  $b_2$

Boundary conditions:  $v_1 = 10$

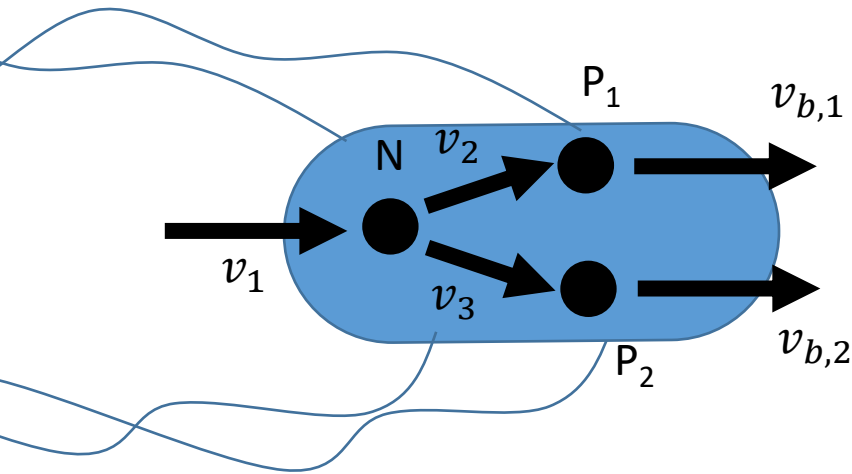
Mass balances:  $v_1 - v_2 - v_3 = 0$

$$v_2 - v_{b,1} = 0$$

$$v_3 - v_{b,2} = 0$$

Biomass function:  $v_{b,1} + 2v_{b,2} = v_{bio}$

# How Does LP work: 2D example



Boundary conditions:

$$v_1 = 10$$

Mass balances:

$$v_1 - v_2 - v_3 = 0$$

$$v_2 - v_{b,1} = 0$$

$$v_3 - v_{b,2} = 0$$

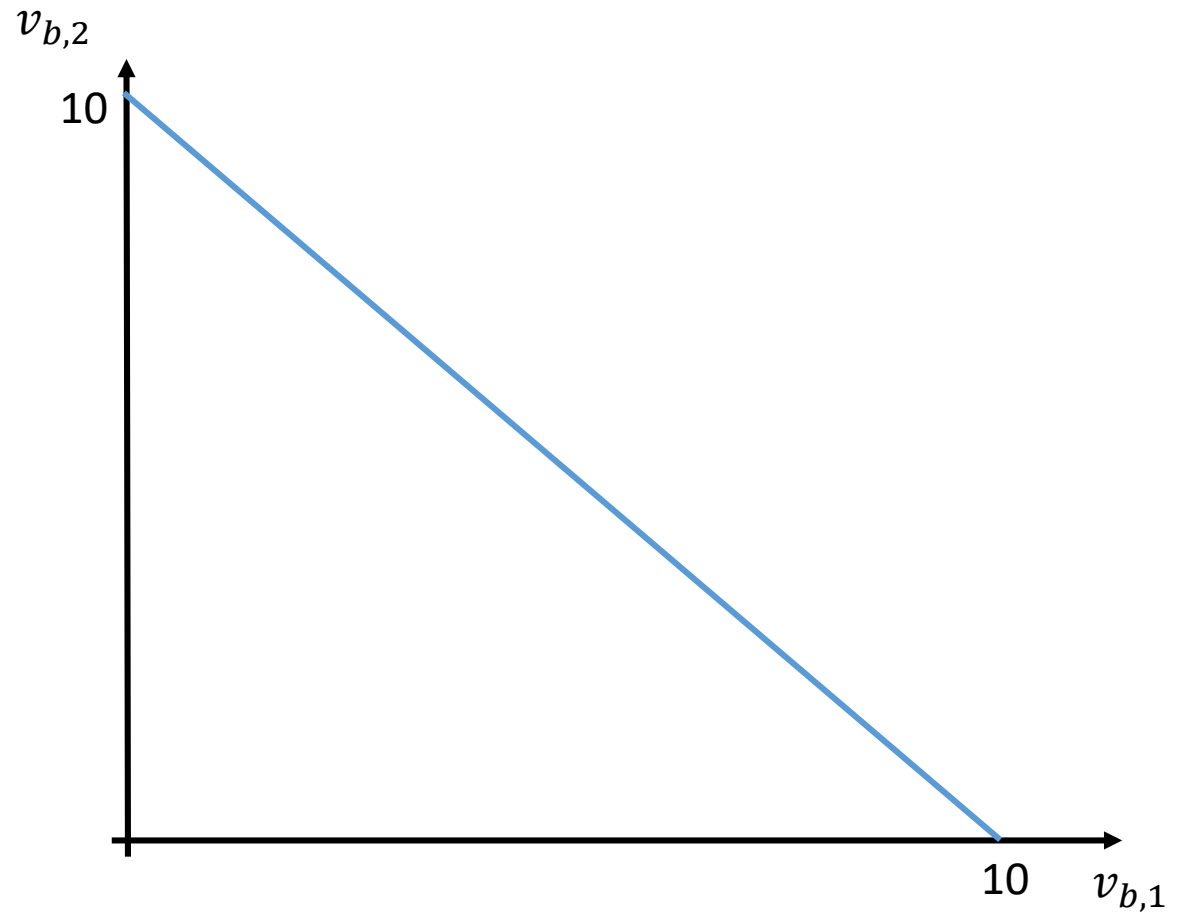
$$10 - v_{b,1} - v_{b,2} = 0$$

Biomass function:  $v_{b,1} + 2v_{b,2} = v_{bio}$

# How Does LP work: 2D example

Equality constraints:

$$v_{b,1} = 10 - v_{b,2}$$



# How Does LP work: 2D example

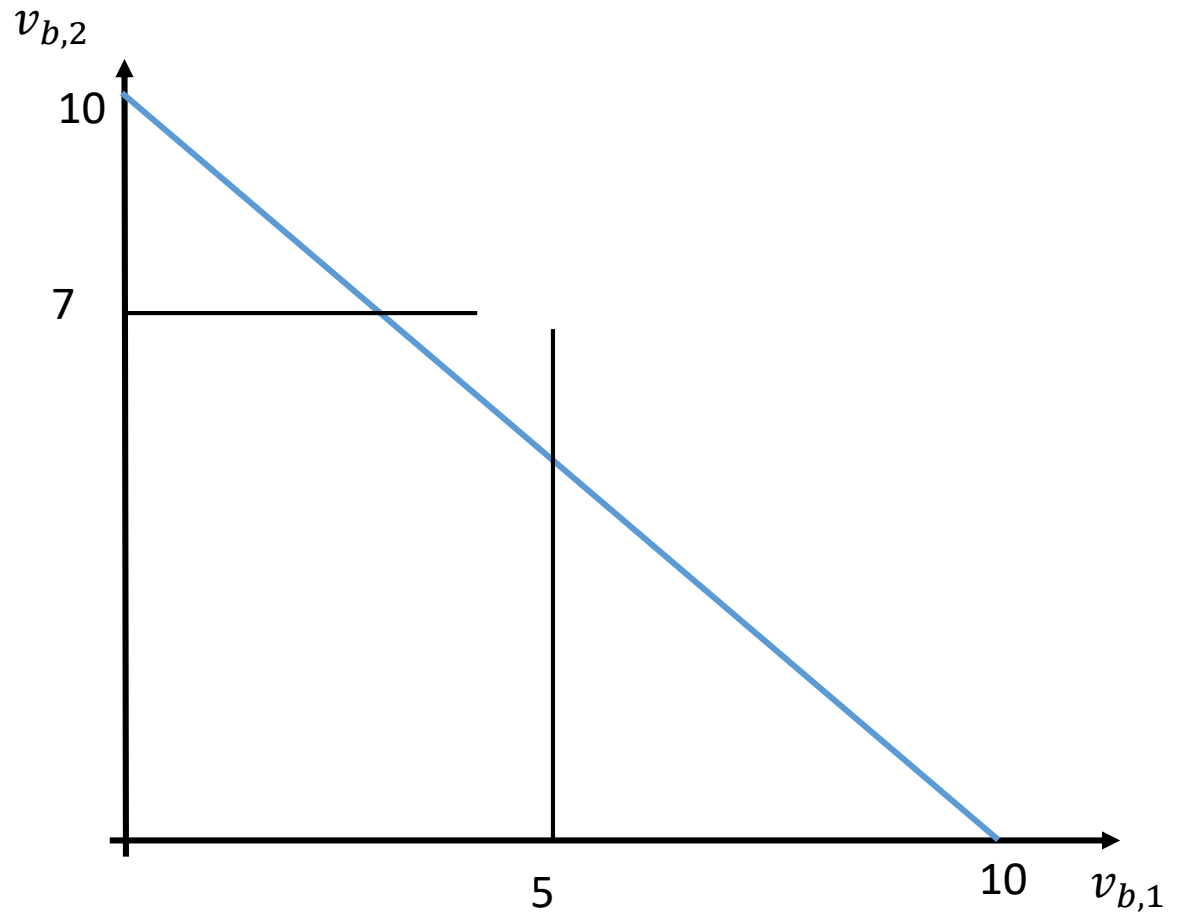
Equality constraints:

$$v_{b,1} = 10 - v_{b,2}$$

Inequality constraints:

$$0 \leq v_{b,2} \leq 7$$

$$0 \leq v_{b,1} \leq 5$$



# How Does LP work: 2D example

Equality constraints:

$$v_{b,1} = 10 - v_{b,2}$$

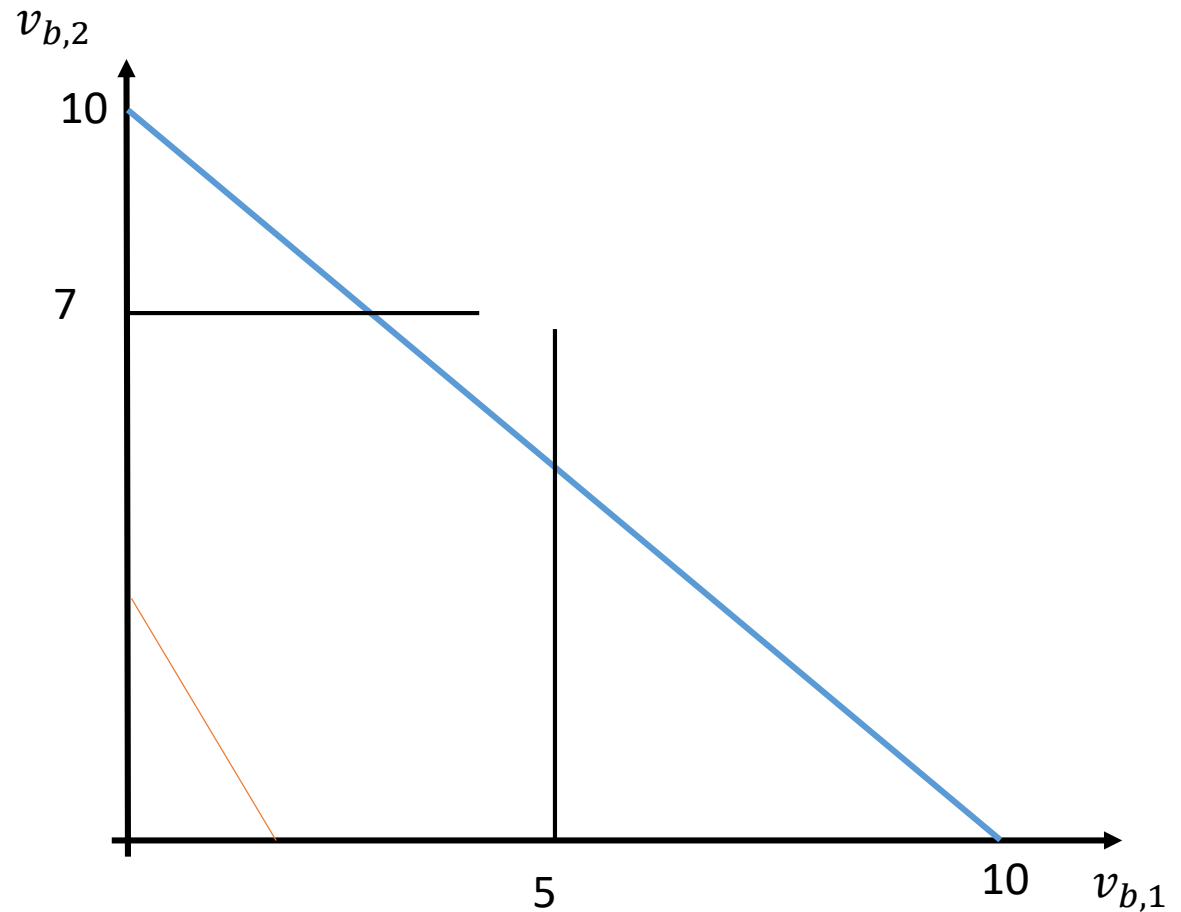
Inequality constraints:

$$0 \leq v_{b,2} \leq 7$$

$$0 \leq v_{b,1} \leq 5$$

Objective function:

$$v_{b,1} + 2v_{b,2} = v_{bio}$$



# How Does LP work: 2D example

Equality constraints:

$$v_{b,1} = 10 - v_{b,2}$$

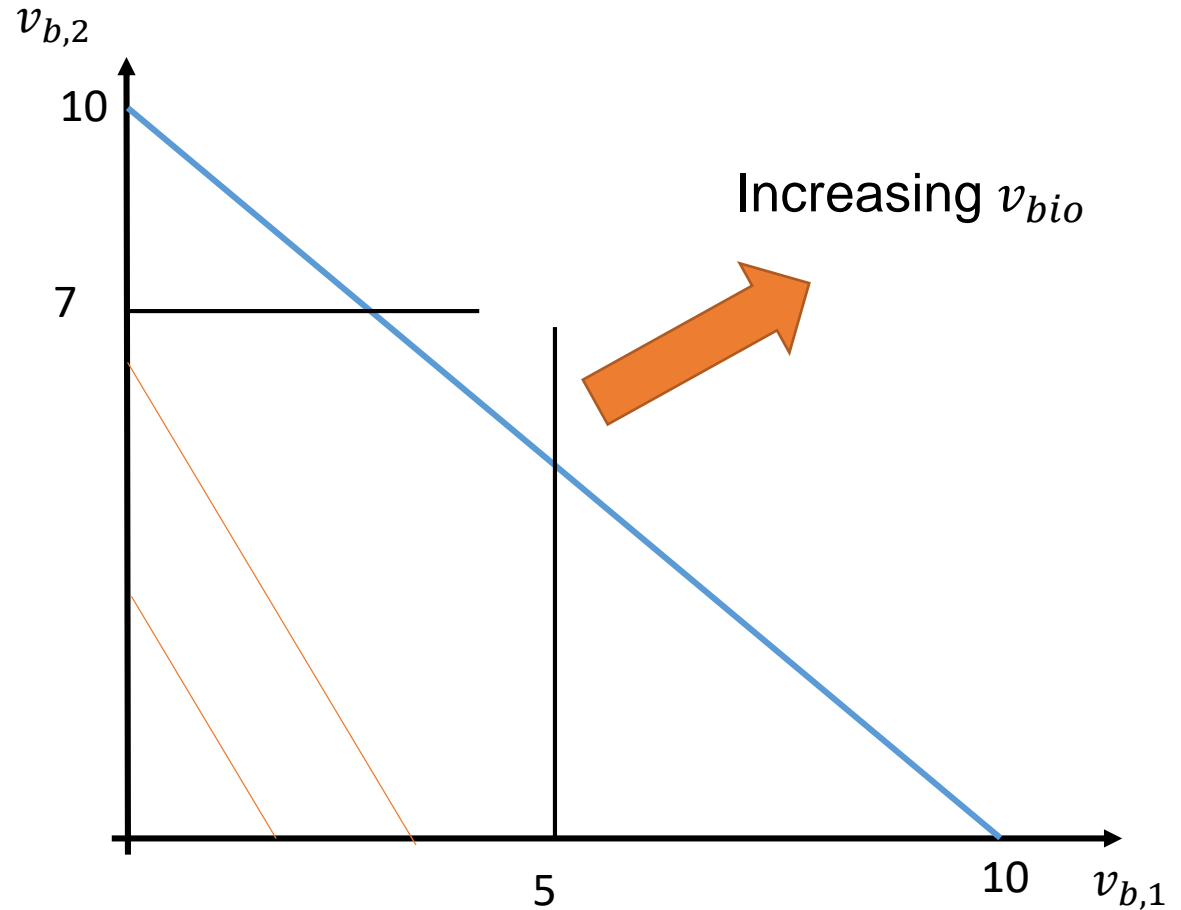
Inequality constraints:

$$0 \leq v_{b,2} \leq 7$$

$$0 \leq v_{b,1} \leq 5$$

Objective function:

$$v_{b,1} + 2v_{b,2} = v_{bio}$$



# How Does LP work: 2D example

Equality constraints:

$$v_{b,1} = 10 - v_{b,2}$$

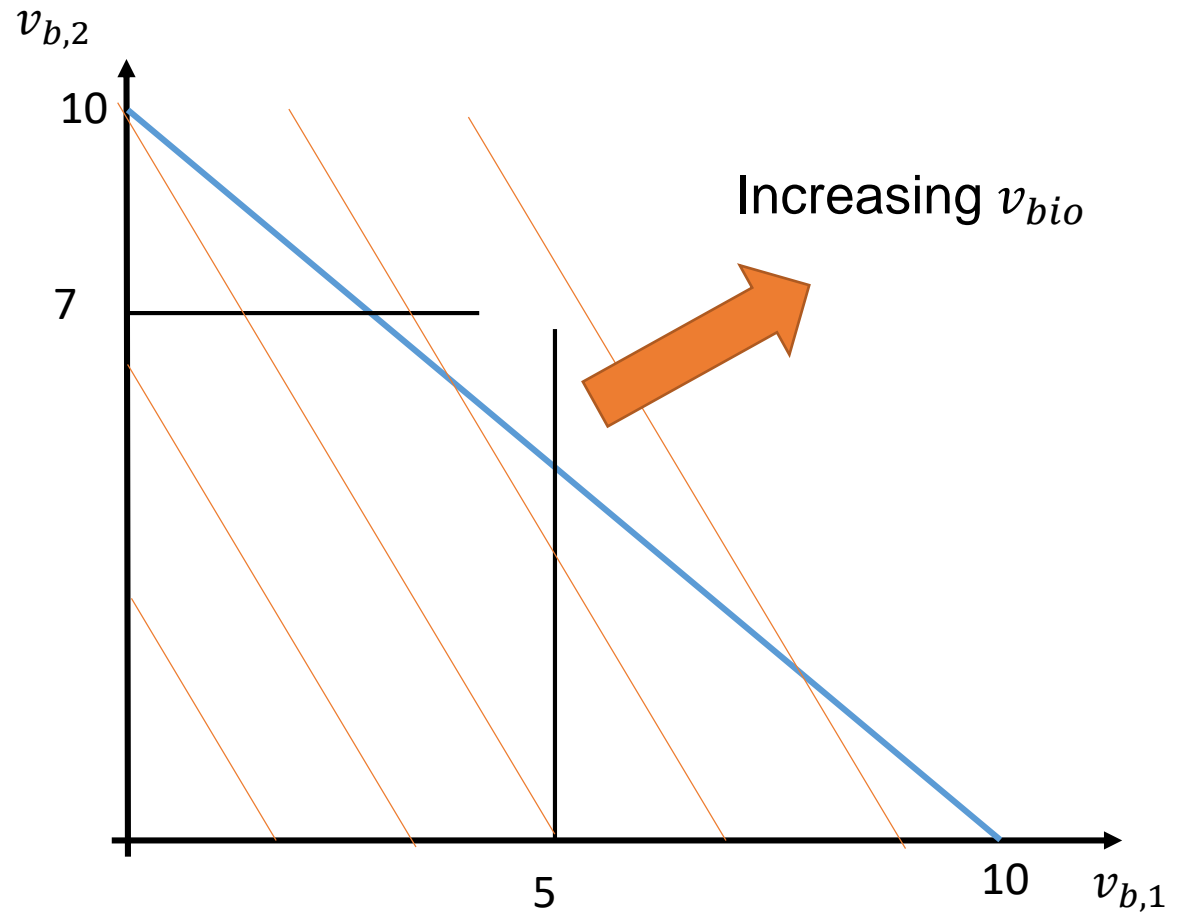
Inequality constraints:

$$0 \leq v_{b,2} \leq 7$$

$$0 \leq v_{b,1} \leq 5$$

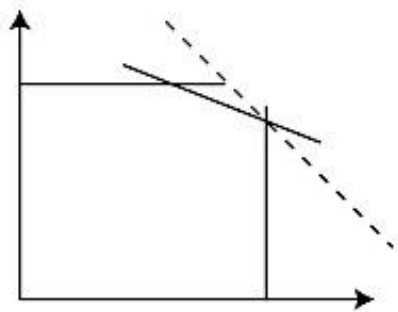
Objective function:

$$v_{b,1} + 2v_{b,2} = v_{bio}$$



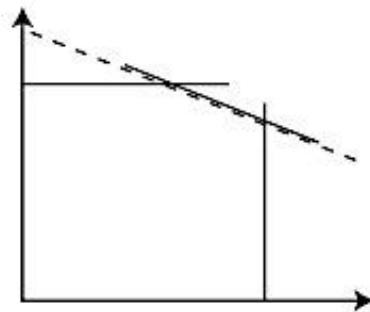
# Types of Feasible Solutions found by LP

Unique solution



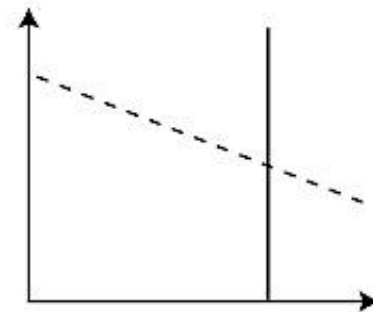
Optimal solution  
in a corner

Degenerate solution



Optimal solution  
along an edge

Unbounded solution

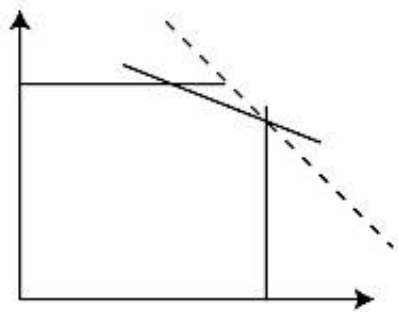


Optimal solution not  
found--region unbounded

----- Lines of constant Z

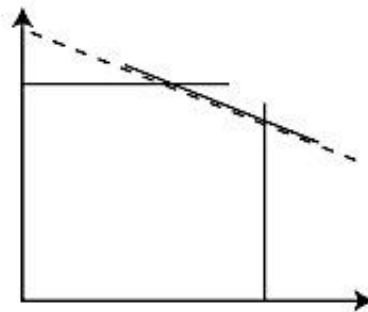
# Types of Feasible Solutions found by LP

Unique solution



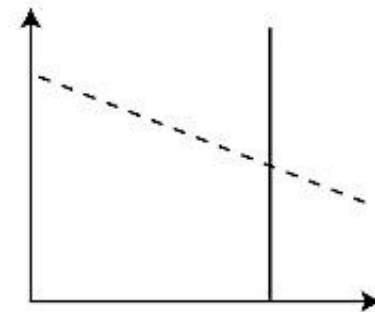
Optimal solution  
in a corner

Degenerate solution



Optimal solution  
along an edge

Unbounded solution



Optimal solution not  
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----- Lines of constant Z

Equivalent Optimal Solutions

**Equivalent optimal solutions** occur frequently in genome-scale networks. Since, Genome-scale networks are typically able to achieve the same overall functional network state in many different ways.

# Flux Balance Analysis

Flux balance analysis (FBA) applies defined **constraints** on an **objective function** and find the optimum solutions (flux profiles)

**Table II.** Questions that can be addressed using flux-balance analysis.

Question	Objective	Reference
<i>What are the biochemical production capabilities?</i>	Maximize metabolite product	Varma, Boesch, & Palsson, 1993
<i>What is the maximal growth rate and biomass yield?</i>	Maximize growth rate	Varma & Palsson, 1993; Varma & Palsson, 1994b
<i>How efficiently can metabolism channel metabolites through the network?</i>	Minimize the Euclidean norm	Bonarius et al., 1996
<i>How energetically efficient can metabolism operate?</i>	Minimize ATP production or minimize nutrient uptake	Majewski & Domach, 1990; Savinell & Palsson, 1992; Fell & Small, 1986
<i>What is the tradeoff between biomass production and metabolite overproduction?</i>	Maximize biomass production for a given metabolite production	Varma et al., 1993

For a given growth condition (e.g. known input nutrients), considering:

- metabolic system operates in a **quasi-steady state**.
- certain **constraints** on system (flux limitations, stoichiometric and reversibility constraints ).
- an “objective” that is expected to be maximized (e.g. **biomass** production).

FBA **predicts** reaction fluxes and essential enzymes under a given growth condition

# Other biological objectives

**Table III** Objective functions implemented in constraint-based FBA

Objective function <sup>a</sup>	Mathematical definition	Explanation	Rationale	Reference
Max biomass <sup>b</sup>	$\max \frac{V_{\text{biomass}}}{V_{\text{glucose}}}$	Maximization of biomass yield	Evolution drives selection for maximal biomass yield ( $Y_{X/S}$ )	(van Gulik and Heijnen, 1995; Edwards and Palsson, 2000b; Price <i>et al.</i> , 2004)
Max ATP	$\max \frac{V_{\text{ATP}}}{V_{\text{glucose}}}$	Maximization of ATP yield	Evolution drives maximal energetic efficiency ( $Y_{\text{ATP}/S}$ )	(van Gulik and Heijnen, 1995; Ramakrishna <i>et al.</i> , 2001)
Min $\sum v_i^{2c}$	$\min \sum_{i=1}^n v_i^2$	Minimization of the overall intracellular flux	Postulates maximal enzymatic efficiency for cellular growth (analogous to minimization of the Euclidean norm)	(Bonarius <i>et al.</i> , 1996; Blank <i>et al.</i> , 2005a)
Max ATP per flux unit <sup>c</sup>	$\max \frac{V_{\text{ATP}}}{\sum_{i=1}^n v_i^2}$	Maximization of ATP yield per flux unit	Cells operate to maximize ATP yield while minimizing enzyme usage	(Dauner and Sauer, 2001)
Max biomass per flux unit <sup>c</sup>	$\max \frac{V_{\text{biomass}}}{\sum_{i=1}^n v_i^2}$	Maximization of biomass yield per flux unit	Cells operate to maximize biomass yield while minimizing enzyme usage	
Min glucose	$\min \frac{V_{\text{glucose}}}{V_{\text{biomass}}}$	Minimization of glucose consumption	Evolution drives selection for most efficient usage of substrate	(Oliveira <i>et al.</i> , 2005)
Min reaction steps <sup>c</sup>	$\min \sum_{i=1}^n y_i^2, y_i \in \{0, 1\}$	Minimization of reaction steps	Cells minimize number of reaction steps to produce biomass	(Melendez-Hevia and Isidoro, 1985)
Max ATP per reaction step <sup>c</sup>	$\min \frac{V_{\text{ATP}}}{\sum_{i=1}^n y_i^2}, y_i \in \{0, 1\}$	Maximization of ATP yield per reaction step	Cells operate to maximize ATP yield per reaction step	
Min redox potential <sup>d,e</sup>	$\min \frac{\sum^n V_{\text{NADH}}}{V_{\text{glucose}}}$	Minimization of redox potential <sup>f</sup>	Cells decrease number of oxidizing reactions thus conserving their energy or using their energy in the most efficient way possible	(Knorr <i>et al.</i> , 2007)
Min ATP production <sup>d,e</sup>	$\min \frac{\sum^n V_{\text{ATP}}}{V_{\text{glucose}}}$	Minimization of ATP producing fluxes <sup>g</sup>	Cells grow while using the minimal amount of energy, thus conserving energy	(Knorr <i>et al.</i> , 2007)
Max ATP production <sup>d,e</sup>	$\max \frac{\sum^n V_{\text{ATP}}}{V_{\text{glucose}}}$	Maximization of ATP producing fluxes <sup>h</sup>	Cells produce as much ATP as possible	(Heinrich <i>et al.</i> , 1997; Ebenhoeh and Heinrich, 2001; Knorr <i>et al.</i> , 2007)

<sup>a</sup>Both maximization of biomass objectives (absolute and per flux unit) require *no a priori* assumptions. For all other objectives the specific growth rate was set to the experimentally determined value under each condition.

<sup>b</sup>Often also referred to as optimization of growth rate (Price *et al.*, 2004).

<sup>c</sup> $n$  refers to the overall number of reactions in the network, that is 98 in the present case.

<sup>d</sup>Reaction name is that specified in Supplementary Table I; ‘\_R’ refers to the reverse reaction.

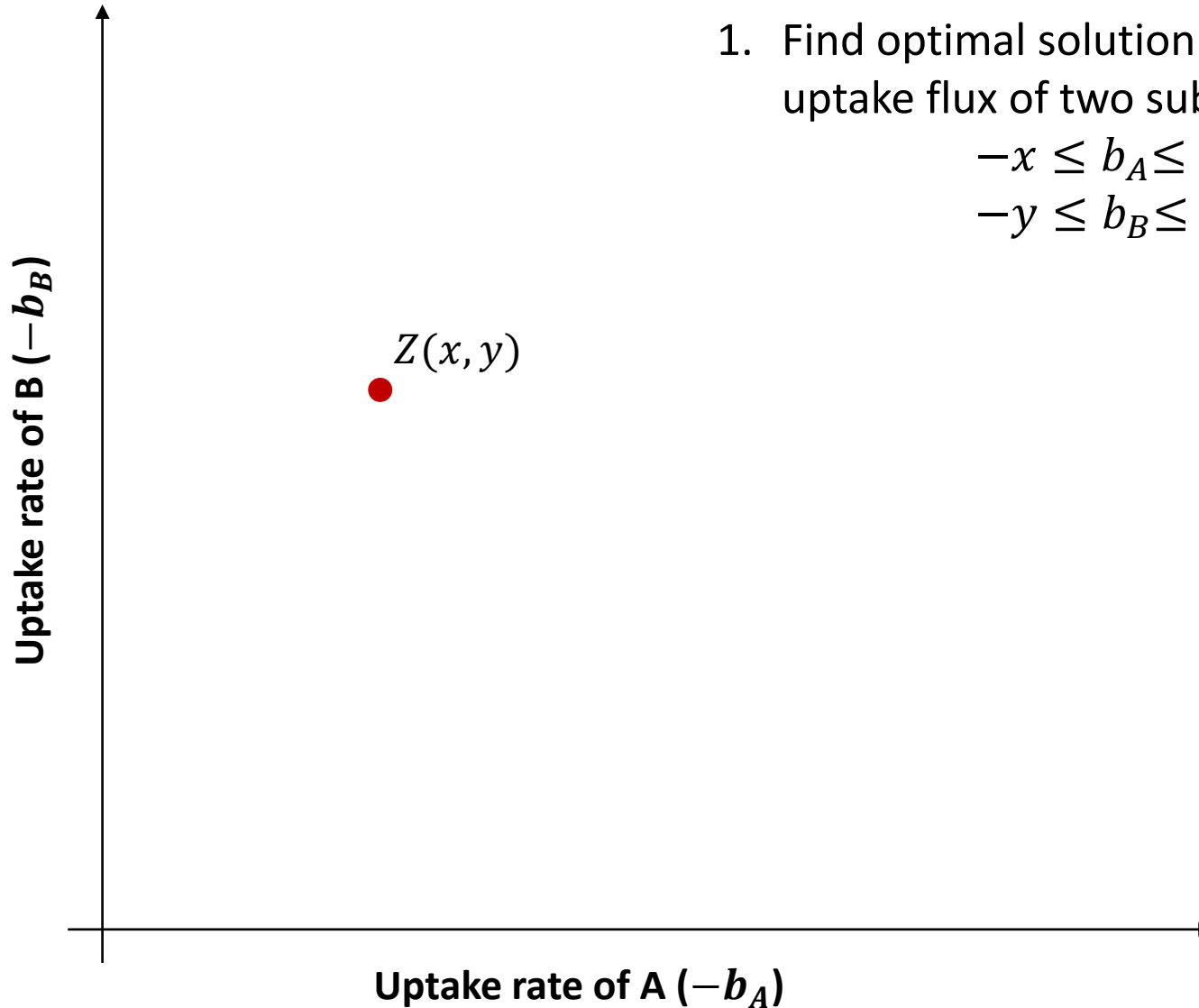
<sup>e</sup>All reversible reactions in Supplementary Table I were converted to two irreversible reactions resulting in a final stoichiometric model of 60 metabolites and 151 reactions.

<sup>f</sup>Reactions: *gapA*, *aceE/F*, *maeA*, *sucAB*, *mdh*, *udhA*, *fdhF*, *fdnGHI*, *fdnGHI*, *ldhA*, *adhE\_R*, *mhpF\_R*, *adhP\_R*, *adhC\_R*, *maeB*, *zwf*, *gnd*, *icd*, *pntAB*, *frdABCD*, *sdhAB*, *dld*, *sdhABCD\_R*.

<sup>g</sup>Reactions: *pgk*, *pykA*, *pykF*, *sucCD*, *atpA-H*, *ackA*, *ackB*, *tdcD*, *purT*.

<sup>h</sup>Reactions: *pgk*, *pykA*, *pykF*, *sucCD*, *atpA-H*, *ackA*, *ackB*, *tdcD*, *purT*.

# Phenotypic phase planes

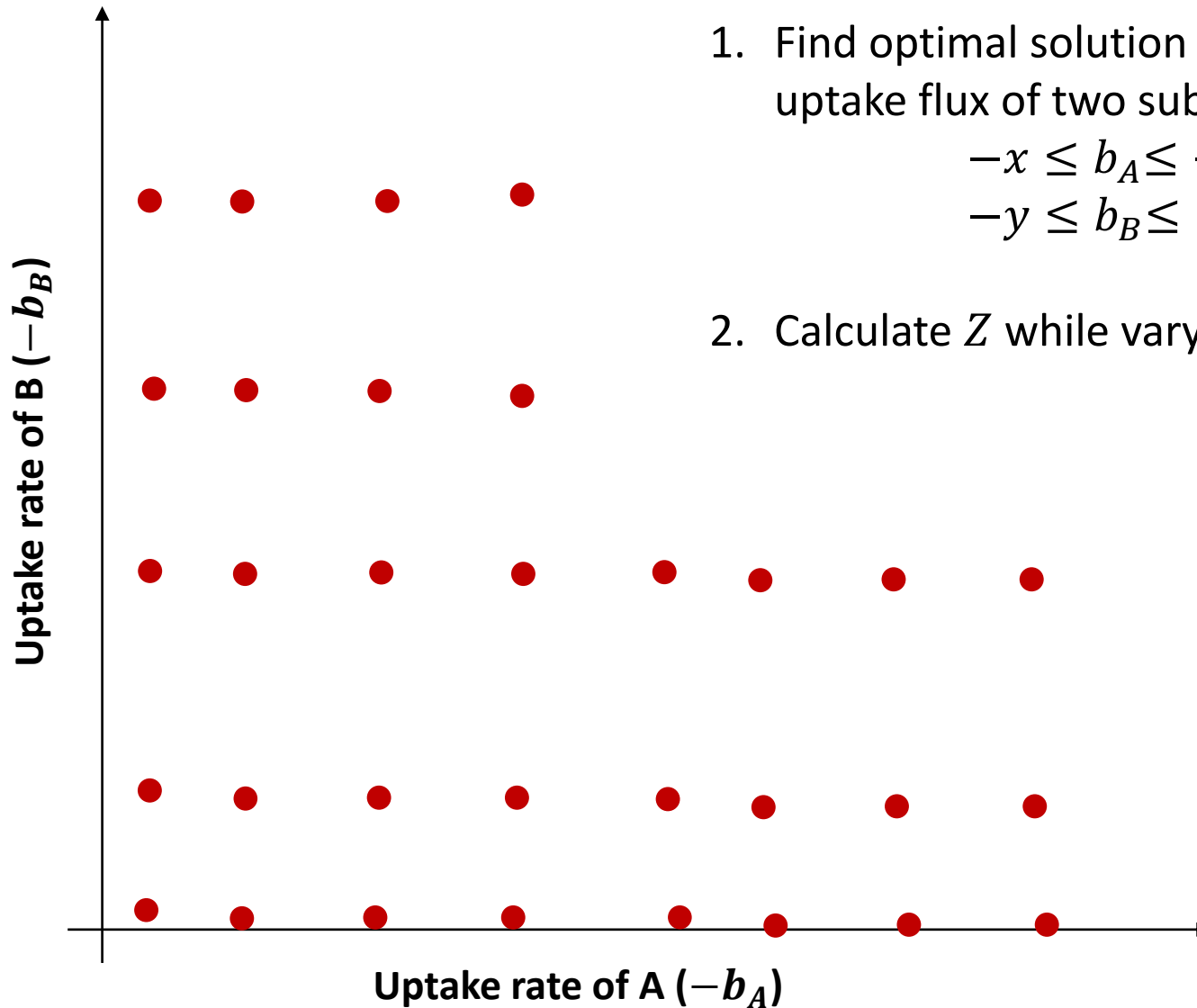


1. Find optimal solution for a fixed uptake flux of two substrates ( $b_A$  and  $b_B$ )

$$-x \leq b_A \leq -x$$

$$-y \leq b_B \leq -y$$

# Phenotypic phase planes



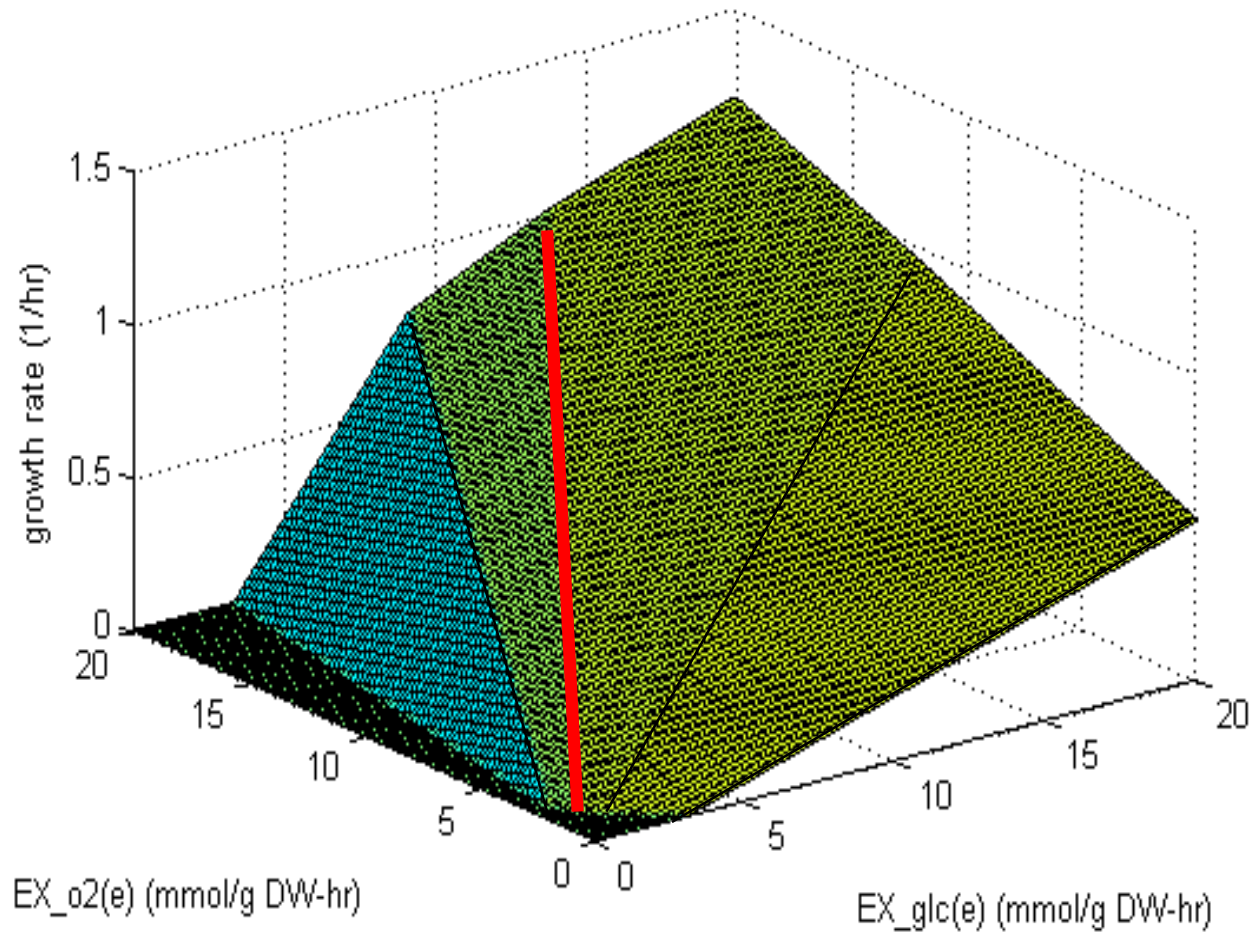
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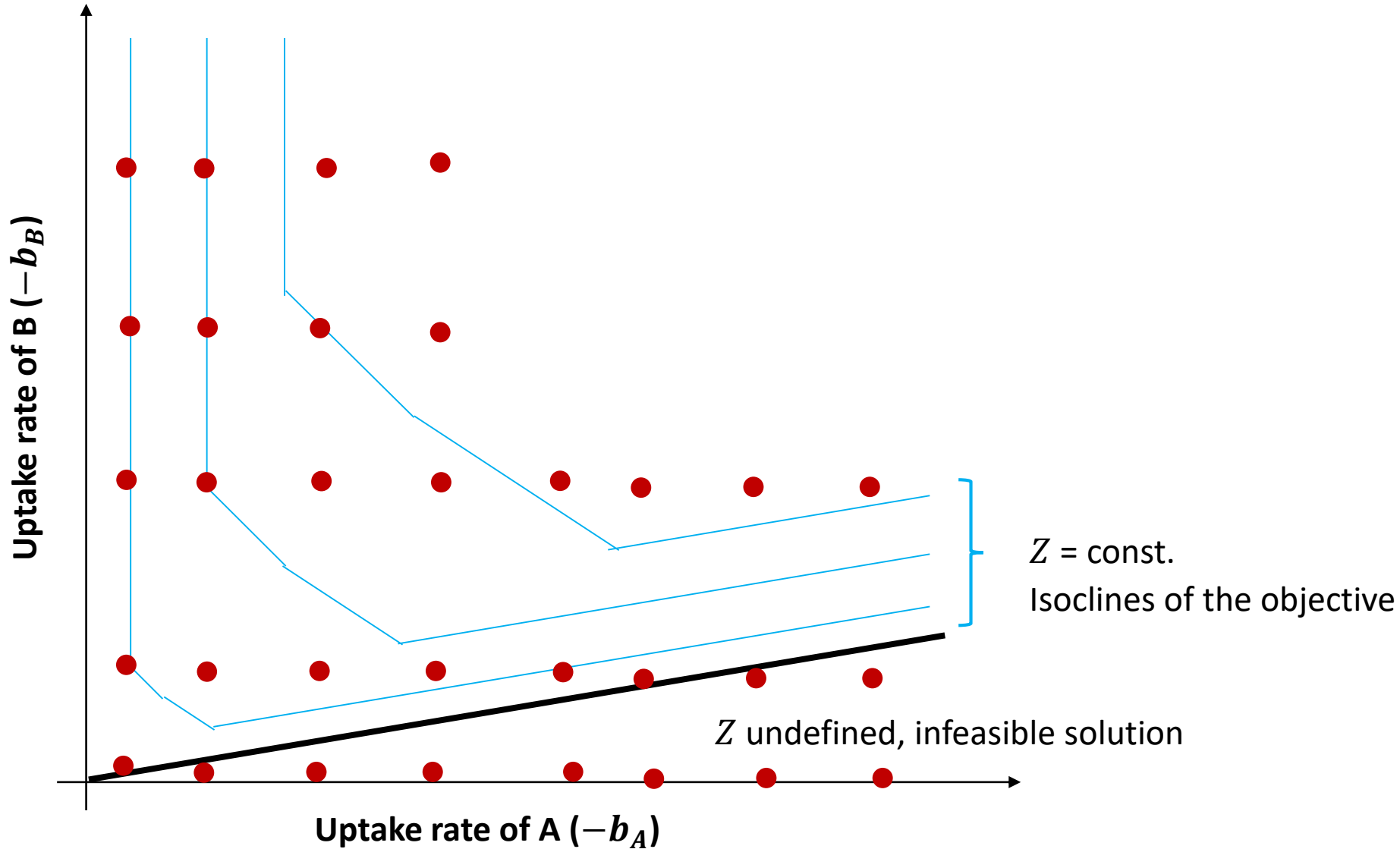
$$-y \leq b_B \leq -y$$

2. Calculate  $Z$  while varying  $x$  and  $y$

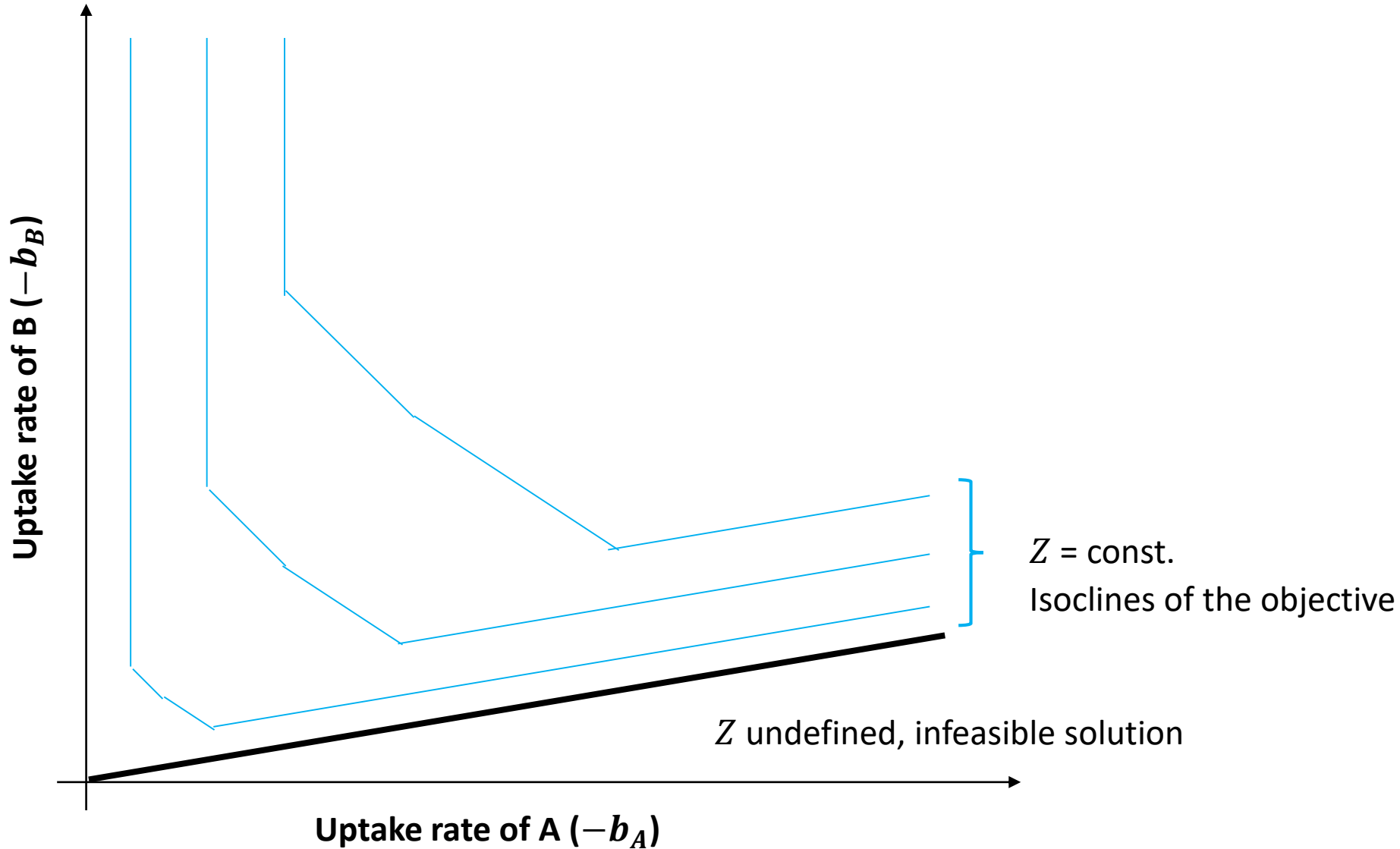
# Phenotypic phase planes



# Phenotypic phase planes



# Phenotypic phase planes



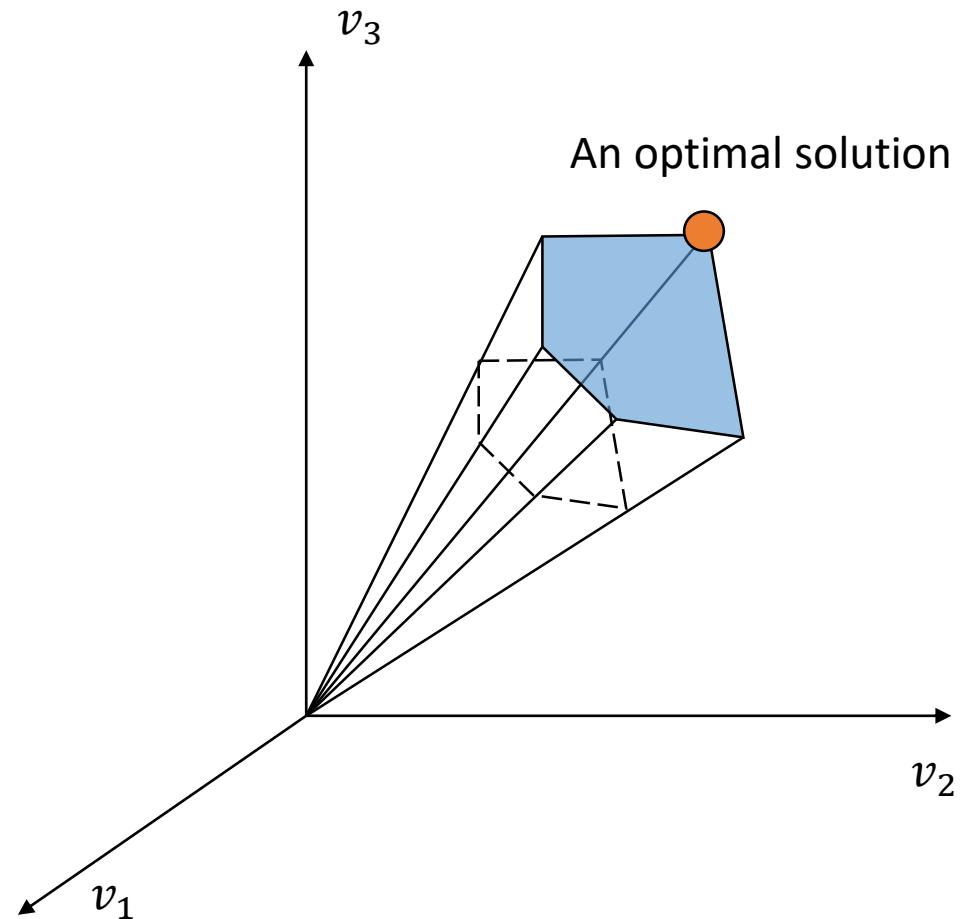
# Flux Variability Analysis

# Flux Variability Analysis

1) Perform an **FBA** with a given objective e.g:

$$S\vec{v} = \vec{0} \text{ with } v_{j,lb} \leq v_j \leq v_{j,ub}$$

$$b_{max} = \max(\text{biomass})$$



# Flux Variability Analysis

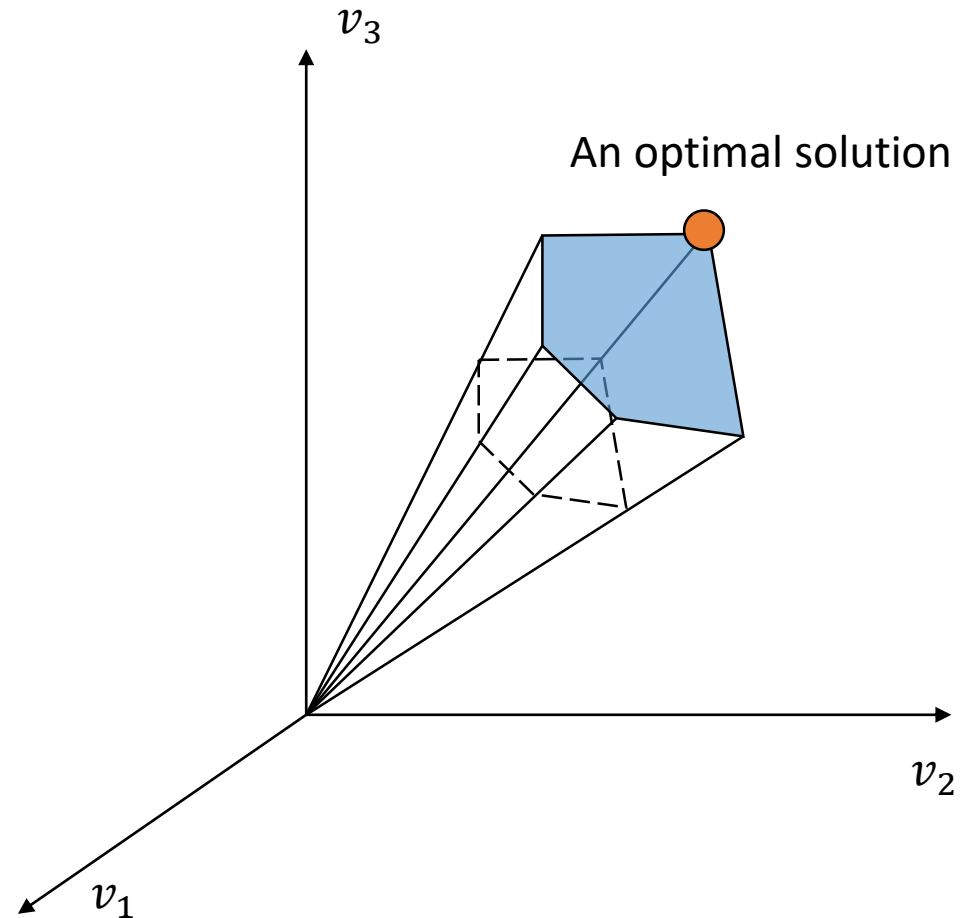
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2) **Constraint** the **objective** e.g:

$$b_{max} \leq \text{biomass}$$

3) Find **minimum** and **maximum** of every flux given the **constraint** of the **optimal objective**:

for  $v_i$  in  $\vec{v}$ :

$S\vec{v} = \vec{0}$  with:

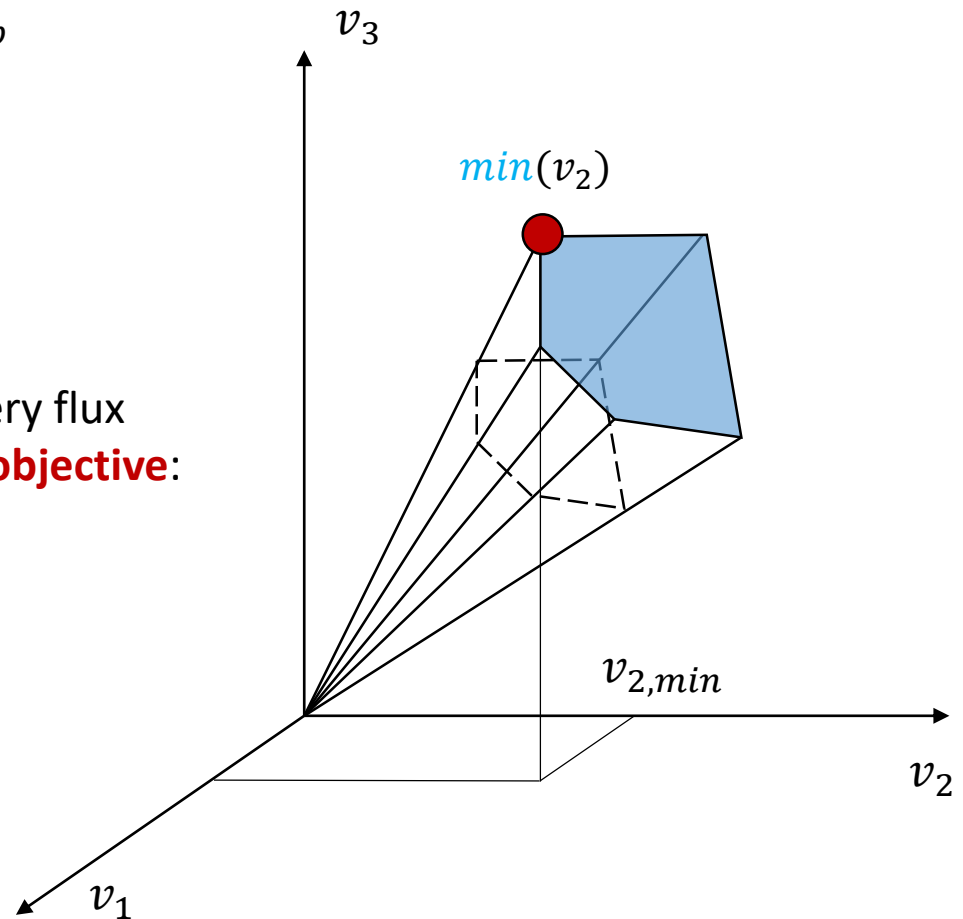
$$v_{j,lb} \leq v_j \leq v_{j,ub} \text{ and}$$

$$v_{max} \leq \text{biomass}$$

$$v_{i,min} = \text{min}(v_i)$$

$$v_{i,max} = \text{max}(v_i)$$

end



# Flux Variability Analysis

- 1) Perform an **FBA** with a given objective e.g:

$$S\vec{v} = \vec{0} \text{ with } v_{j,lb} \leq v_j \leq v_{j,ub}$$

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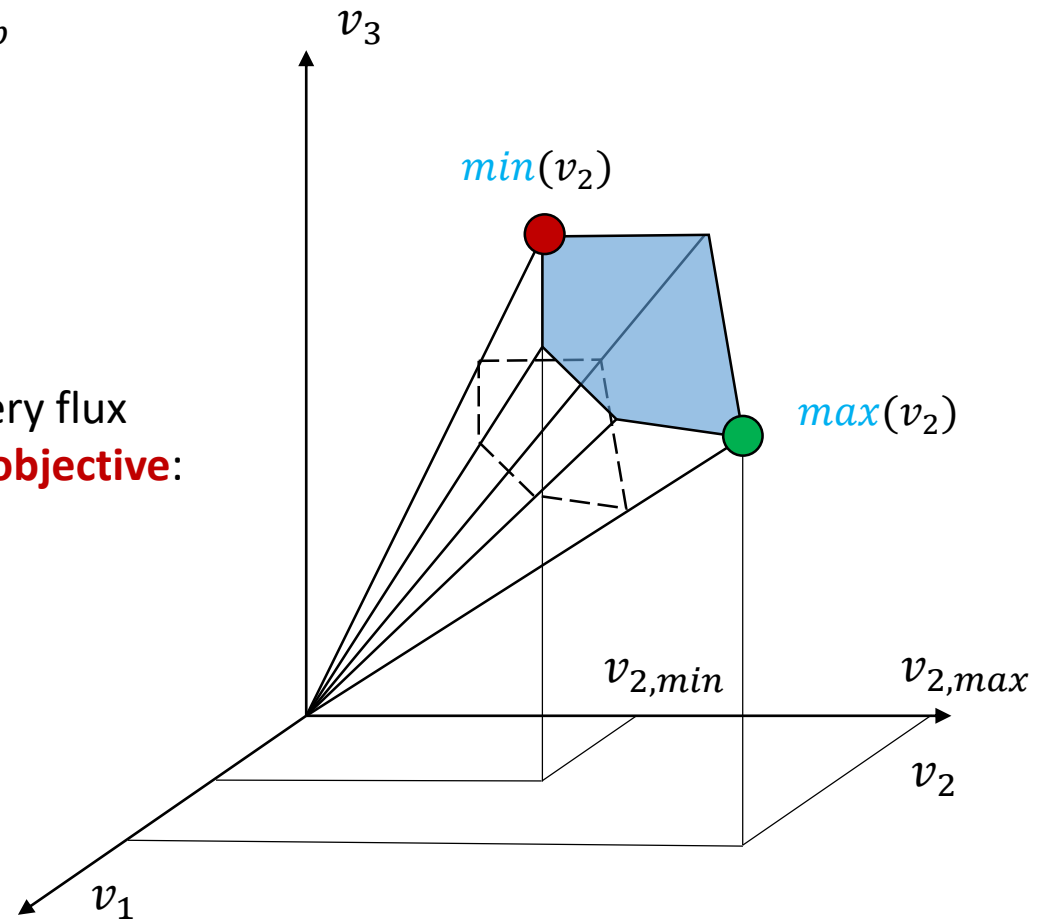
$$v_{j,lb} \leq v_j \leq v_{j,ub} \text{ and}$$

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$$v_{i,min} = \text{min}(v_i)$$

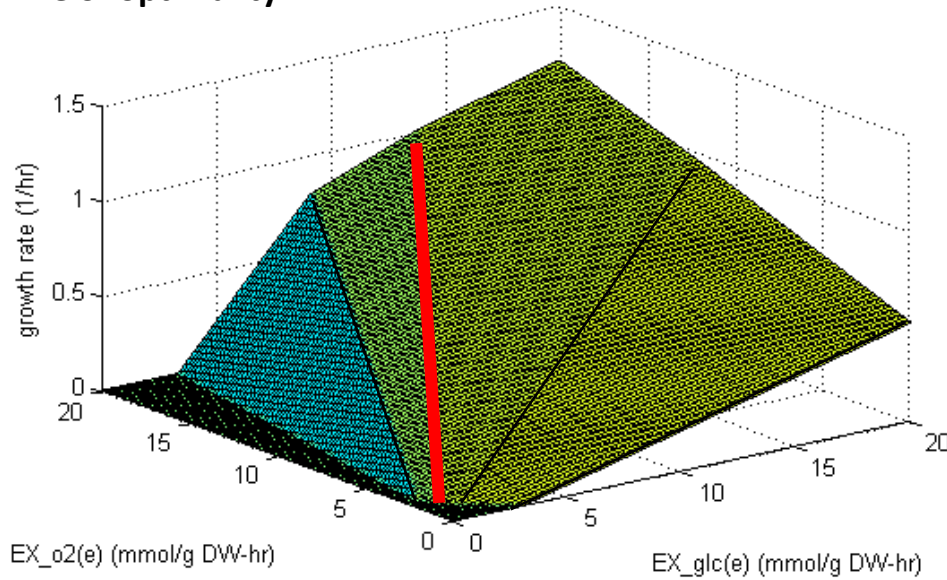
$$v_{i,max} = \text{max}(v_i)$$

end



# Phenotypic phase planes

## Line of optimality

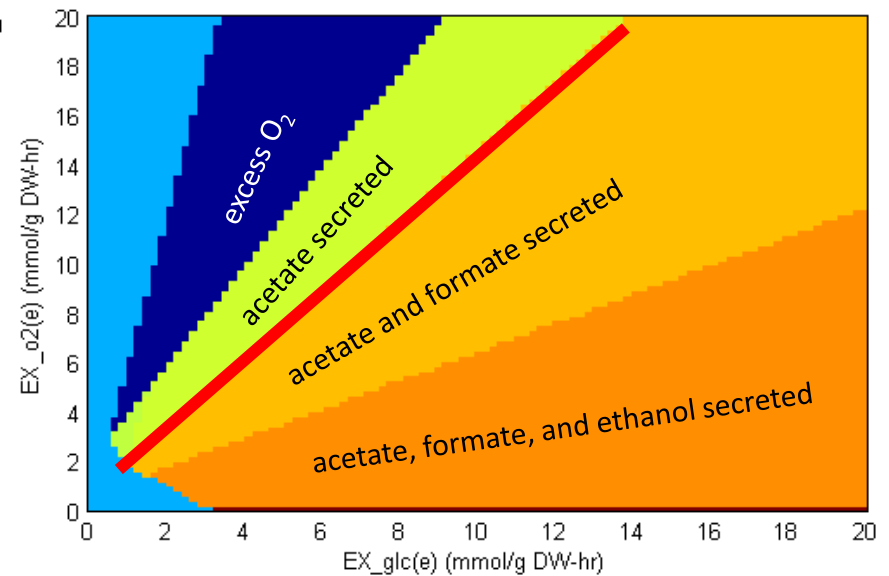


## Line of optimality:

Maximal biomass yield with respect to a carbon source (no oxygen limitation!)

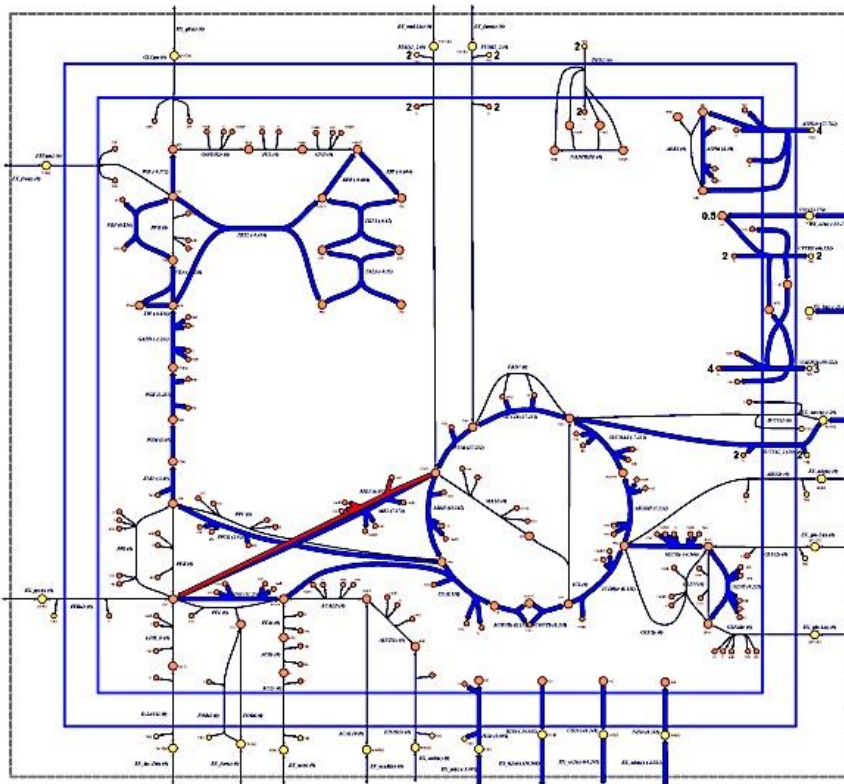
Line of optimality

## Line of optimality

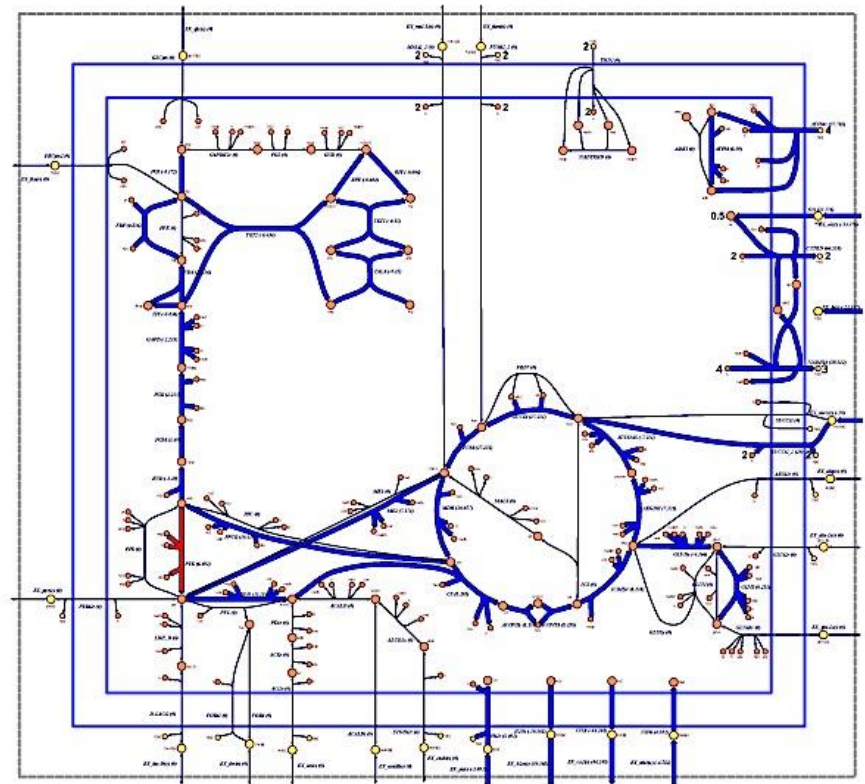


# Alternative flux profiles

a



b



Reaction	Minimum Flux (mmol gDW <sup>-1</sup> hr <sup>-1</sup> )	Maximum Flux (mmol gDW <sup>-1</sup> hr <sup>-1</sup> )
FRD7	0	972.77
MDH	13.56	20.06
ME1	0	6.49
ME2	7.17	13.67
NADTRHD	0	6.49
PPCK	3.93	10.42
PYK	0	6.49
SUCDi	27.23	1000

Two alternate solutions for **maximum aerobic growth** on succinate.

a) Reaction **ME1** is used to convert **L-malate** to **pyruvate**

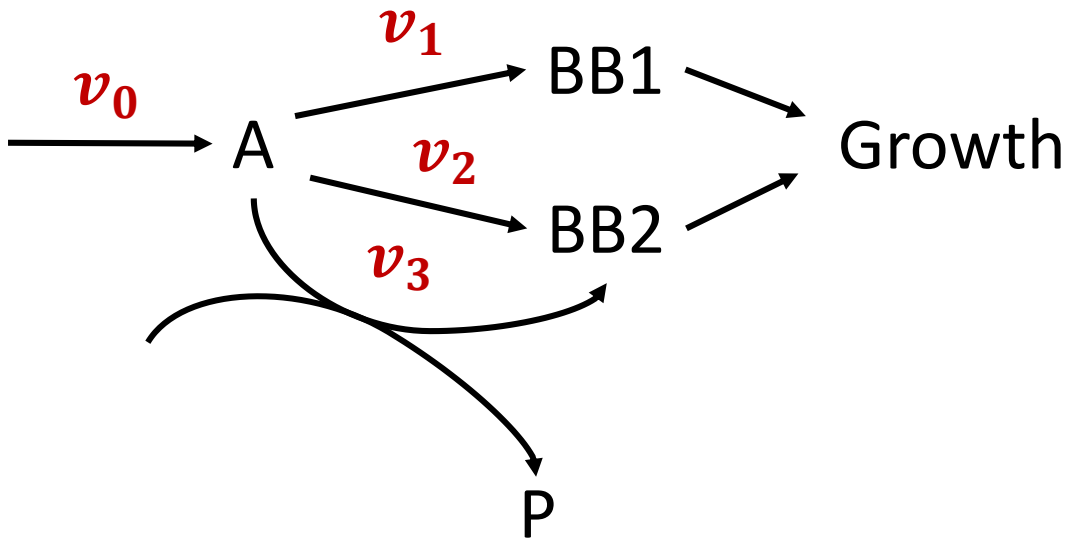
b) The reaction **PYK** is used to perform this function.

The two alternative reactions are highlighted in red.

# Alternative flux profiles

Where do they come from?

$$v_0 = 3 \frac{\text{mmol}}{\text{gDW h}}$$

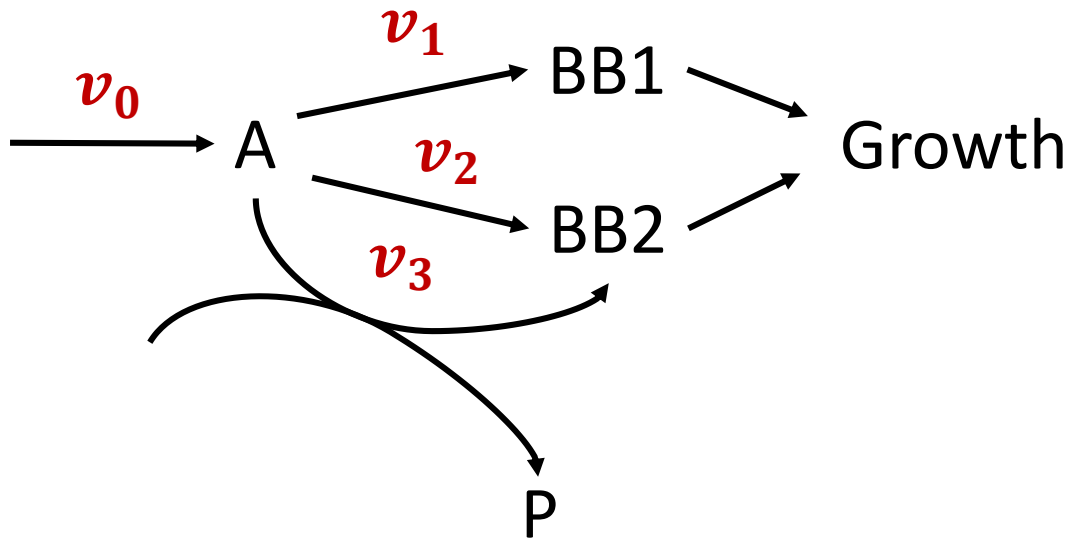


Flux Variability		
Reaction	Min Flux	Max Flux
Growth		

# Alternative flux profiles

Where do they come from?

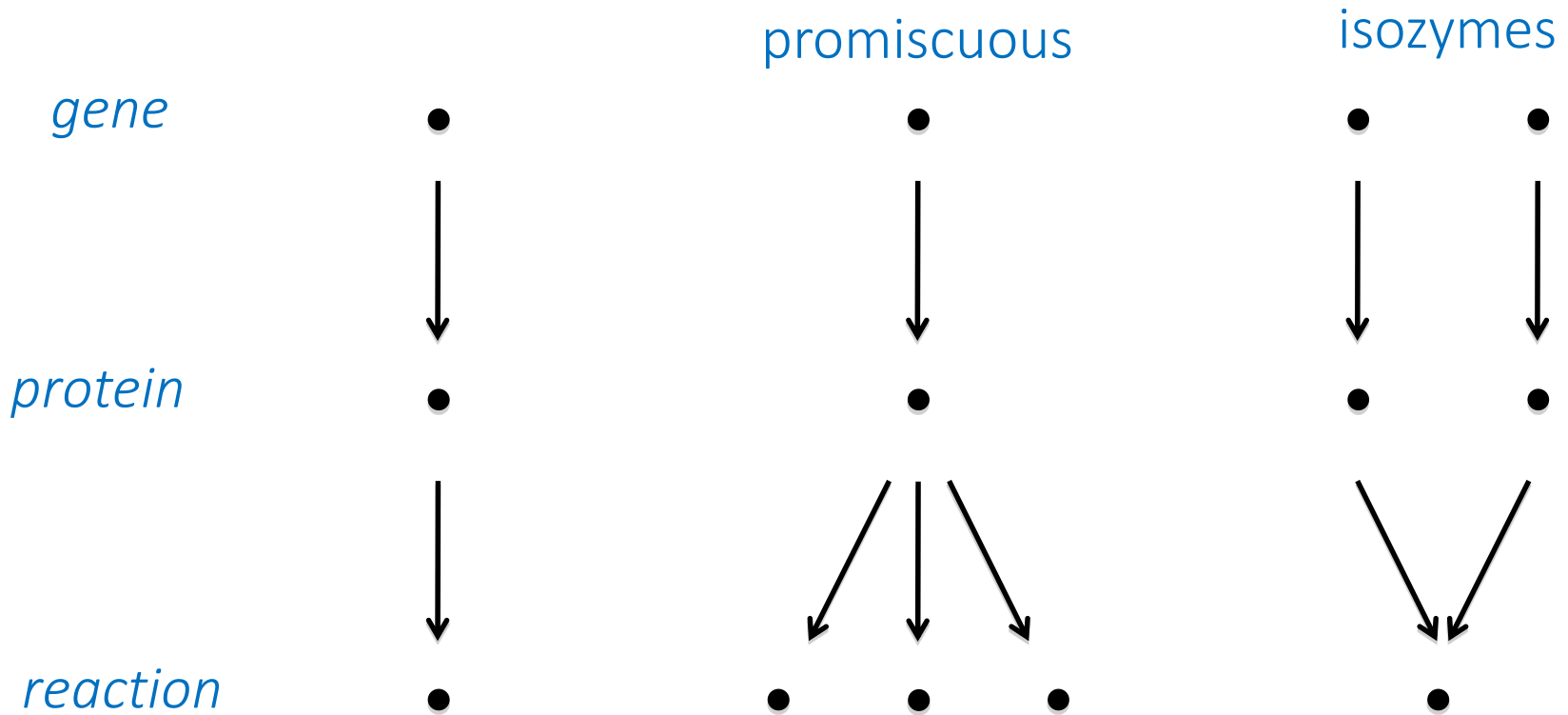
$$v_0 = 3 \frac{\text{mmol}}{\text{gDW h}}$$



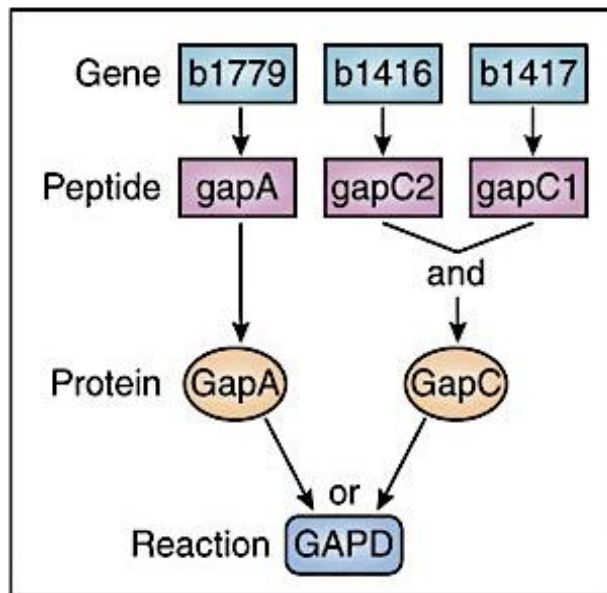
Flux Variability		
Reaction	Min Flux	Max Flux
	1.5	1.5
	0	1.5
	0	1.5
Growth	1.5	1.5

# Gene Essentiality

# From genes to function



# From genes to function

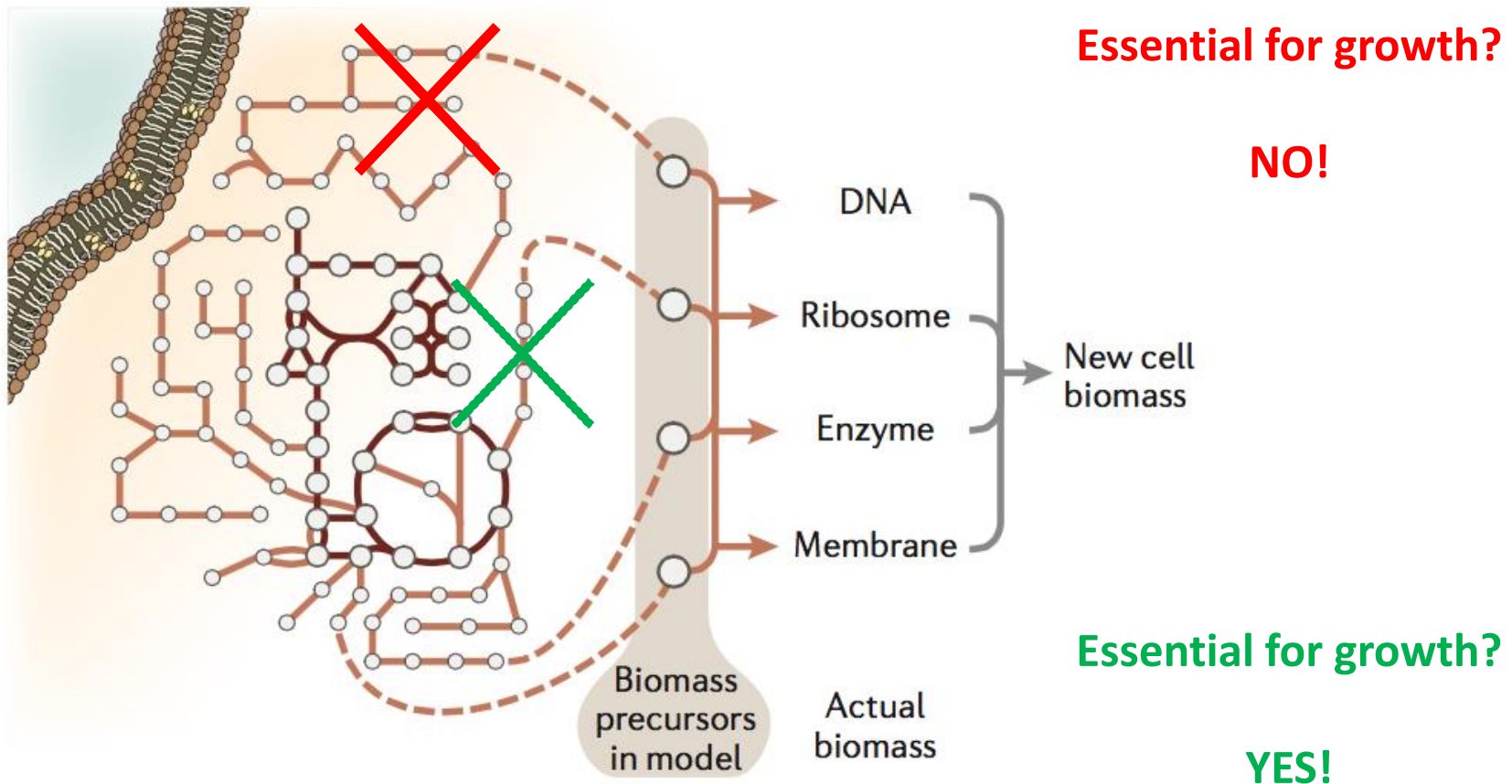


Abbreviation	Glycolytic reactions
HEX1	$[c]GLC + ATP \rightarrow G6P + ADP + H$
PGI	$[c]G6P \leftrightarrow F6P$
PFK	$[c]ATP + F6P \rightarrow ADP + FDP + H$
FBA	$[c]FDP \leftrightarrow DHAP + G3P$
TPI	$[c]DHAP \leftrightarrow G3P$
<b>GAPD</b>	<b><math>[c]G3P + NAD + PI \leftrightarrow 13DPG + H + NADH</math></b>
PGK	$[c]13DPG + ADP \leftrightarrow 3PG + ATP$
PGM	$[c]3PG \leftrightarrow 2PG$
ENO	$[c]2PG \leftrightarrow H_2O + PEP$
PYK	$[c]ADP + H + PEP \rightarrow ATP + PYR$

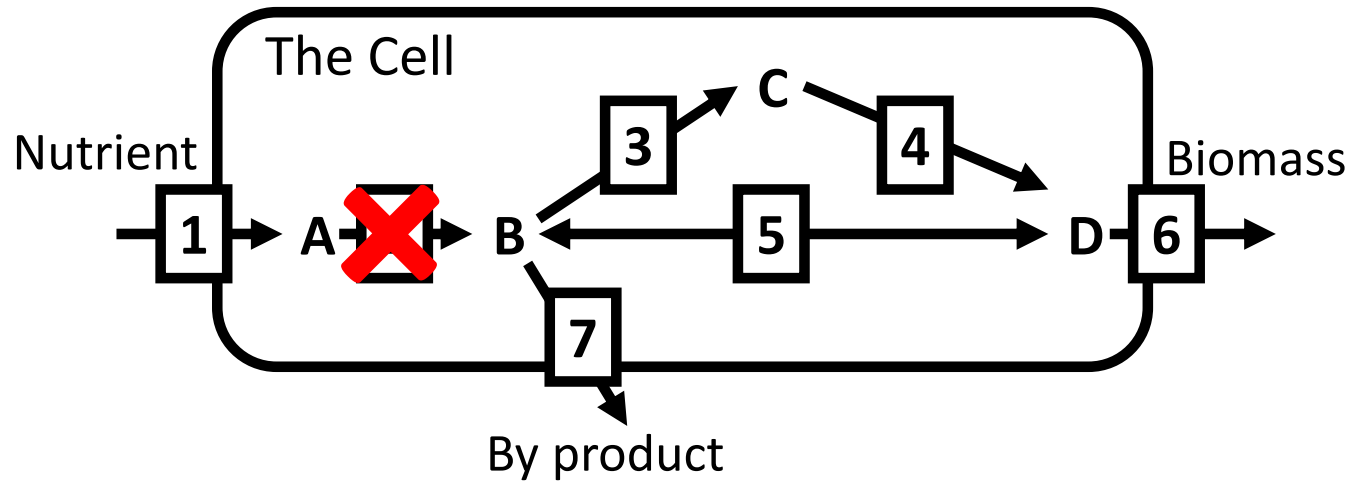
# Knockout Studies -> Gene Essentiality

What makes a gene essential ?

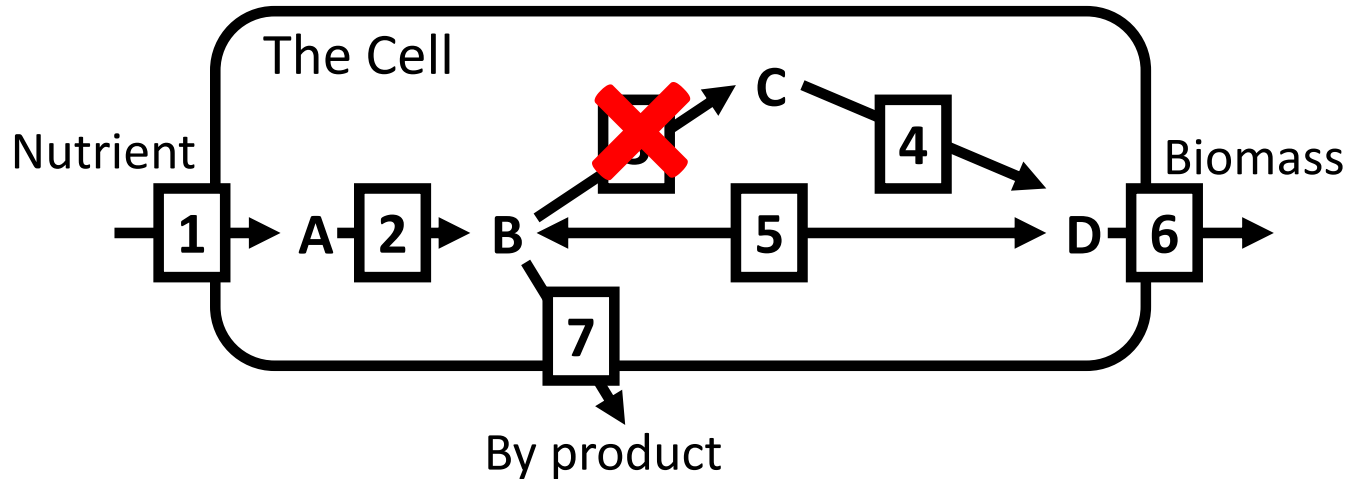
- Biomass precursors required for growth,...



# Example

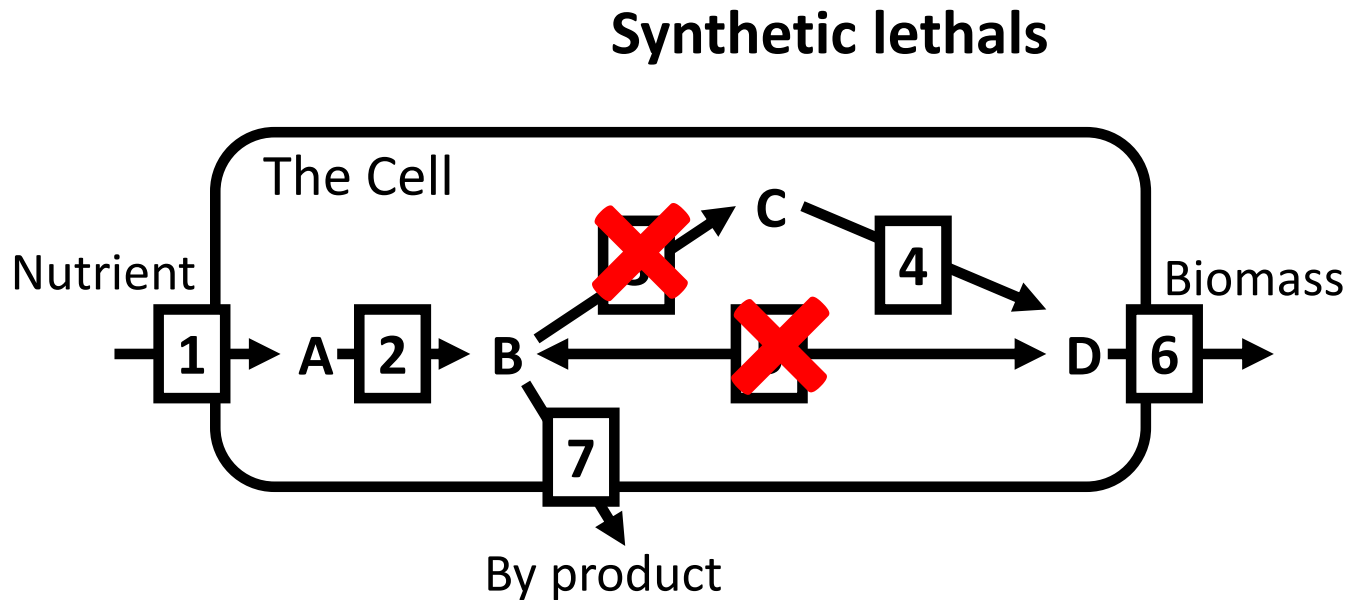


# Example



- **Localization:** gene 3 is localized in the cytosol
- **Function:** gene 3 encodes for enzyme 3 that catalyzes reaction from B to C
- **Lethal effect ?:** No the presence of gene 5 enables the production of molecule D that is required for growth

# Example



- **Localization:** genes 3 & 5 are localized in the cytosol
- **Function:** both encode for enzymes/pathways that enable production of D
- **Lethal effect:** The knockout of genes 3 & 5 impedes the production of molecule D that is required for growth